Modelling multiple long tail liability lines
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1. Introduction and summary

In order to calculate and assess risk metrics for long tail liabilities, a modelling framework that can describe the salient statistical features of the data in a succinct, yet complete, fashion is required. Model specification error is mitigated if simulated triangles from the optimal model are indistinguishable, in respect of the salient statistical features, from the real data.

The framework presented here for modelling multiple loss development arrays (segments or LOBs for example) captures:

- trends in the three time directions (accident, development, and calendar);
- distributions of the volatility around the trends;
- process (volatility) correlation between loss development arrays; and
- parameter correlation within and between loss development arrays.

Modelling or forecast scenario assumptions are transparent, auditable, and verifiable. Case studies are provided which demonstrate the power, flexibility, and wide applicability of this modelling framework to produce creative solutions to long-tail liability risk management.

1.1. Modelling framework

The Probabilistic Trend Family (PTF) modelling framework describes the trends in each direction (development, accident, and calendar) along with the volatility around the trends. Within this framework, an optimal model is identified that captures the trends in the three directions along with the volatility around those trends.

The following diagram depicts the three directions with arguments d, w, and t. Since t = w + d, it is axiomatic that any calendar year trend projects onto the development year and accident year directions.

The Multiple Probabilistic Trend Family (MPTF) modelling framework is an extension of the PTF modelling framework where correlations between LOBs are included to describe the relationship within and between LOBs. These correlations are data driven and are unique to each company’s portfolios. An MPTF model is designed from the identified PTF models for each segment and includes the correlation between the segments or LOBs.

A single composite model can describe all the long-tail liabilities in a company’s portfolio.
The PTF and MPTF models address the individual features that are found in the data – there is no a priori specification where trends (or volatility) changes occur. Trends are fully interpretable and are able to be related to events occurring in the business (whether driven by internal or external drivers). Each cell of the loss development array is related by the trend structure on a log scale. Correlations between the distributions, whether within or between segments (or LOBs), are incorporated directly in the model.

1.2. Wealth of information in ICRFS-PLUS™

All the ICRFS-PLUS™ tables and graphic displays based on the identified (optimal) composite model, in the MPTF modelling framework for multiple LOBs (or segments), can be replicated in matter of seconds as a result of Insureware’s extremely fast computational algorithms. ‘What if’ analyses can be considered and results obtained very quickly.

One double click loads the identified model and reveals pictorially the volatility structure of each long tail LOB and their inter-relationships (correlation structures). Critical financial information including reserve distributions by accident year, calendar year and total for each LOB and the aggregate of all LOBs, reserve distribution correlations between LOBs, risk capital allocation by LOB and calendar year, TV@Rs and V@Rs for different time horizons, and more can be computed approaching real time (seconds). A company-wide report for long-tail liability lines can be created with a single report template.

The identified model fits correlated log-normal distributions to each observation in the data triangles. Similarly, correlated log-normal distributions are projected for each future cell. Complete control is retained over future assumptions (in any direction) including parameter uncertainty (and volatility about trends); all assumptions are explicit and auditable. The MPTF modelling framework provides a sound and solid statistical foundation for conducting ‘what if’ scenarios in respect of reserving, pricing, Solvency II and IFRS 4 analyses, reinsurance analyses, and many other applications.

The probabilistic framework initialises data updates with the prior model structure. This provides effortless monitoring of stability and an early warning system for changes in trends.

1) Forecast distributions for each future cell, for each segment (or line of business) for any aggregation across segments.
2) Reserve forecast distribution correlations between LOBs by total, accident year, and calendar year.
3) Summary tables by accident year, including one-year ahead statistics (equivalently, variation in mean ultimates one-year hence).
4) Summaries by calendar year.
5) Risk capital allocation by calendar year and accident year.
6) Graphs of ultimates versus accident year and future liability stream versus calendar year.
7) Summaries by Line of Business; means and CVs.
8) Aggregate distributions by accident year (simulated from predicted correlated log-normals), calendar year, and total - including Value-at-Risk (V@Rs) and Tail-Value-at-Risk (TV@Rs) - for each segment and any aggregation.
9) Economic balance sheet that includes: Solvency II risk metrics, risk capital calculations and graphs.
10) Distributions for the aggregate, and each segment, for future underwriting (accident) years used for pricing.

The following information, critical for the calculation of Solvency II metrics, is supplied:

• Probability distributions of the paid losses by calendar year \((k=1,\ldots,n)\), and their correlations for each LOB and the aggregate of all LOBs where complete run-off is achieved at the ultimate calendar year \(n\).
• Probability distributions of the paid losses conditional on the first calendar year’s losses being at the 99.5th percentile; that is, the year is ‘in distress’ with a 1/200 year event.
The MPTF modelling framework provides the required distributions. Thus, any risk measure can be computed, including Value-at-Risk (V@R) for calendar year k, for each LOB, and the aggregate of all LOBs.

ICRFS-P furnace contains the unique PTF and MPTF modelling frameworks. Data, models, forecast scenarios, and links to reports all reside in a relational database. The database is a repository for all triangle groups (containing triangles, premiums, exposure measures, models and reports etc.) indexed by line of business, group member, territory and/or any other user-defined criteria.

1.3. Case studies

The remainder of this document consists of a series of case studies illustrating the depth and breadth of applications of the ICRFS-PLUS™ MPTF modelling framework for long-tail liability risk management.

Section 2: Company M

Company M consists of three Lines of Business split into various segments and cost components. The MPTF modelling framework is used to connect the models for the individual cost components. Results are presented in aggregate (all Lines of Business) as well as by line and cost component – all using the optimal single composite model identified in the MPTF modelling framework.

This study showcases the ICRFS-PLUS™ MPTF modelling framework and covers the following topics:

- Forecast scenarios encompassing various aggregations across cost components and LOBs;
- Complete reserve distribution analysis,
  - Allocation by Line of Business and Segment;
  - Summaries by accident year (including variation in mean ultimates one year hence);
  - Summaries by calendar year;
  - Quantiles (percentiles), V@R and T-V@R tables;
- Future underwriting years distributions and risk metrics;
- Combined (reserve and future underwriting years) distributions, V@Rs and T-V@Rs, and risk diversification;
- Economic Balance Sheet, Solvency II and IFRS4 metrics,
  - One-year risk horizon,
  - Ultimate year risk horizon, and
- Reserve releases based on conservative forecast scenarios and monitoring.

Section 3: Companies A and B: Credibility modelling

Company A and B are both casualty treaty syndicates. Company A’s data are sparse compared to Company B, resulting in some critical trend parameters not being able to be estimated with a good level of precision. Some of these parameters are estimated by pooling strength from Company’s A data (and model). That is, Company’s A data is used to credibility adjust the model for Company B. Process volatility is not credibility adjusted as it is (typically) intrinsic to the LOB. The identified MPTF composite model for Company A and B is used to forecast distributions for each company and the aggregate.

Section 4: Company S: Losses and Recoveries

As with Company M, Company S writes a large portfolio of Motor and other LOBs. LOBs are split into various loss development array components. For the two Motor LOBs, recoveries also apply. The MPTF modelling framework is used to model both losses and recoveries. Loss distributions are presented net of recoveries by subtracting the projections for recoveries from the losses.

Section 5: A.M. Best Schedule P: BH, SR, and the Industry

In this section, two companies, BH and SR, are compared using A.M. Best Schedule P data. Calendar year trends in the company’s LOBs are shown to be unique and distinct from each other and the industry.

Section 6: Worker’s Compensation Segments: SAD and SAM

These two segments of Worker’s Compensation are shown to be closely related in accident year trend structure. The mean ultimates move synchronously. There is a functional relationship in mean ultimates that is approximately linear. This relationship is stronger than volatility correlation.

The process (volatility) correlation, after the trends are adjusted for, is only 0.25 and reserve distribution correlation is only 0.086! Relationships in mean ultimates are not process correlation - see also “Understanding correlations and common drivers” brochure.

In respect of forecasting future accident (underwriting) years, the relationship between the accident year trends is important. Selection of future assumptions must be cognisiant of this relationship.
2. **Company M: Reserve, Solvency II, Risk capital allocation, pricing future years, and more**

This case study illustrates the following topics in the context of Company M – a large company writing Motor and Professional Liability Insurance:

- Correlation: process (volatility), parameter and reserve distribution
- Combinations of forecasts to create summaries in aggregate, by line of business, and by cost component
- Risk capital allocation: by line of business, by calendar year, or by accident year
- Complete reserve, future underwriting year, and combined (reserve + future underwriting year distributions.)
- Monitoring and releasing reserves
- Solvency II metrics
- Combined, reserve, and future underwriting risk

Company M consists of three lines of business: Light Auto, Heavy Auto and Professional Liability. These three lines are split into several segments and cost components as detailed below. When modelling the cost components, eighteen paid loss arrays comprise the composite model.

From the identified composite model at the component level, aggregates at any upper layer can easily be created. A composite model is designed for the whole company’s long tail liabilities with a complete view into any layer and cost component.
2.1. Model displays

A probabilistic trend family (PTF) model is identified for each loss component. The PTF model describes the trends in the three directions (development, accident, and calendar) along with the volatility around the trends. The identified PTF models are then connected via correlations measured from the data in the Multiple Probabilistic Trend Family (MPTF) framework.

The model displays for the eighteen cost components are summarised below, grouped by LOB. All model displays are laid out as detailed in the following graph.

The trends (development, accident, and calendar) and volatility are unique to each component. Relative to exposure adjustment, economic inflation may be expected to be common between the components, however it is clear that social inflation is very different (calendar year trends are social + economic inflation).

Light Auto
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Heavy Auto

The model displays for each component show different development year, accident year, and calendar year trends for each piece. Some cost component’s development trends decrease almost immediately (Light Auto: A:PL(I), Heavy Auto: A:ME, Prof Liab. D&O), other cost components do not decay many years into the payment stream (Light Auto: A: Other, Heavy Auto: A: Other, PL B:PL(I)).

Similarly, calendar year trends vary significantly across cost components and between the three Lines of Business.

In general, the model identified for each component is statistically optimal for the cost component. However, statistically insignificant trends can be included in a model (eg: HA: B PL(I)) where the actuary believes these trend assumptions more appropriately describe the trends in the business.

The critical feature of identifying trends in all three directions, along with the volatility around the trends, allows the actuary to make informed decisions about both trends measured in the model along with indications of future emerging trends.
2.2. Correlations between LOBs
There are three types of correlations between Lines of Business:

- Process (volatility) correlation,
- Parameter correlation, and
- Reserve distribution correlation.

**Process correlation** is the correlation in the pure volatility component of the liabilities. This is measured after all trends have been accounted for.

**Parameter correlation** can arise from the action of external effects, but is also induced via process correlation because estimation of model parameters depends on data subject to correlated random effects. If the trend structure is identical for two lines of business the process correlation will have no impact on parameter uncertainty (this is necessary otherwise data could be duplicated and have lower parameter uncertainty than the original data with no true gain of information).

**Reserve distribution correlations** between calendar year liability streams are a function of process variability and parameter uncertainty; higher parameter uncertainty results in higher reserve distribution correlations.

Correlation should be measured from the data in order to determine each company’s unique interline correlation. Taking these into account results in alterations in parameter and volatility estimates and hence in reserve distributions. These effects cannot be replicated by the imposition of off-the-shelf correlation matrices or copulas. Correlation is an intrinsic component of a good model.

2.2.1. Impact of correlation

Two lines are (positively) correlated when their results tend to consistently miss their target values in the same way. This is what should concern business planners, because it affects the unpredictable component of the forecasts. What is predictable, when it includes common trend patterns, does not count towards correlation, because its effects are already incorporated into the model and forecast. A forecast must include a volatility measure, ideally in the form of a loss distribution but at least in the form of a standard deviation.

Modelling multiple Lines of Business in the Multiple Probabilistic Trend Family modelling framework leads to significant aggregate risk diversification credit. The level of diversification is dependent on the correlations between the Lines of Business.

For further information on correlations, see the “Understanding correlations and common drivers” brochure.
Process correlation

The vast majority of the process correlations are zero (and not displayed). There are several groups of cost components with significant process correlations with another group – these groups are highlighted above. While some process correlations are within LOBs (like LA.A with LA.A:Lost) others are between LOBs (LA.B:Lost with H.A:Other).

The net result is significant diversification credit between LOBs – the process correlations between the three LOBs are very low.

Parameter correlation

Parameter correlations are calculated between all parameters. Below we illustrate an extract of the parameter correlation table for the calendar year (iota) parameters.

Reserve distribution correlations

Reserve distribution correlations are typically much less than process (or parameter) correlations. In fact, reserve distribution correlations are driven by parameter correlation (not process correlation). If parameter correlations are low, then reserve distribution correlations will also be low. These correlations are presented in the section on forecast results.
2.3. Aggregate forecast results

All forecast scenarios presented below correspond to a reasonable scenario which typically continues with, or resumes, positive calendar year trends (and their uncertainties) observed in the data. The specification of future forecast scenario assumptions is entirely within the control of the actuary.

Below multiple aggregations are covered in the tabs:

- All lines is the sum of the LOBs: Light Auto, Heavy Auto, and Prof. Liab. (Professional Liability);
- Aggregate is the sum of the individual cost components;
- All Auto is the sum of Light Auto and Heavy Auto.
- Light Auto and Heavy Auto are the sums of their respective loss components;
- The remaining tabs correspond to projections, by cell and totals, for the respective cost components.

(*) Standard deviations in each cell (red) are standard deviations of log-normal distributions when examining an individual segment (in this case cost component), but are the standard deviation of the sum of correlated log-normal distributions, otherwise. Burgundy standard deviations are always the sum of correlated log-normal distributions.

The mean of the total reserve (reasonable forecast scenario) for the aggregate of all the segments is 1.604B (highlighted: green). The corresponding standard deviation (highlighted: yellow) of the reserve distribution is 115.5M.

The future calendar year payment stream is shown and the comparison between last observed values (blue numbers) and fitted values (black numbers) for the individual cells and calendar year totals 2007-2011 (say) can be made with the projected figures for calendar years 2012-2014. Means are black. The red and burgundy standard deviations correspond to standard deviations of the projected distributions. For the All Lines tab, an aggregate of multiple LOBs, all standard deviations are the standard deviations of the sum of correlated log-normal distributions.

For each cost component, the identified composite model and the associated forecast scenarios project log-normal distributions (and their correlations) for each cell for each segment

The liability stream by calendar year (important for asset-liability matching) is reasonable and consistent with expectations of future losses for this company.

The liability stream means for past calendar years (1999~2011), future years (2012~2031), and the future liability stream means ± one standard deviation (dashed lines) are shown below. The actual losses for the calendar years are marked for 1999~2011.
2.3.1. Reserve forecasts by LOB

Summary statistics for the reserve distributions are provided above for each LOB and the aggregate of the LOBs. The distributions can be further split into each cost component as illustrated in the next section.

2.3.2. Reserve forecasts by cost component

The breakdown of the total reserve to each cost component (grouped by LOB) is shown above. The majority of the total reserve (~71%) is allocated to the two lines: LA.B and LA.A. This is also shown in the pie chart below.
The outer ring (pie chart), shows the aggregation by LOB. The inner pie chart shows the distributions between cost components. Similarly, the coloured bars for the CV(%) on the right, also are grouped by LOB. The CV of the aggregate distribution is only 6.43% while many individual cost components have a CV exceeding 20%.

**Reserve distribution correlations by LOB**

The reserve distribution correlations across the lines of business (not cost components) are very low. Similarly, the reserve distribution correlations are negligible. As such, significant diversification credit is gained from writing the three LOBs. This was evident from the low process correlations illustrated earlier.

The reserve distribution correlations between cost components are predominantly zero with some correlations between lines with process correlation as estimated in the model. The lack of significant process correlation between cost components results in the low correlations between LOBs despite correlations between some cost components being high.
2.3.3. Accident year summaries (including variation in mean ultimates one year hence)

Variation in mean Ultimates conditional on next calendar year’s data

The table above includes the one-calendar-year-ahead mean ultimate statistics (Cond. on Next Cal. Per.). These statistics provide:

- The conditional standard deviation of all possible mean ultimates (one year hence) is given in the rightmost column (+_Ult|Data).
- The second column from the right (Std.Dev.|Data) represents the mean standard deviation of the distribution of the ultimate conditional on next year’s data. The reason it is lower than the standard deviation of the ultimate as at year end 2011 is a result of reduced parameter uncertainty and the forecasting horizon being shorter (when you are at the end of 2012).

The total variability in the ultimate is thus decomposed into the variability for the next calendar year’s forecast (+_Ult | Data) and the variability associated with the remaining years’ forecast (Std.Dev | Data).

Prior to receiving the 2012 data (the next calendar year), the mean of all ultimates conditional on 2012 data is the mean ultimate as at year end 2011.

**Accident year 2011 example**

The projected mean ultimate at the start of 2012 is 354.1M. The variation in mean ultimate for this accident year is 22.7M. This means that the mean ultimate recalculated at the end of 2012 may vary by 22.7M or more from the earlier figure and still be consistent with the estimates above.

The 29.6M corresponds to the standard deviation of the ultimate distribution conditional on the calendar year 2012’s data.

Note: $22.7^2 + 29.6^2 = 37.3^2$ – Pythagoras’ theorem applies.

**Correlations by accident year**

Correlations are calculated between accident period totals for the different aggregations including all lines as well as each cost component.
2.3.4. Calendar year summaries

For the aggregate of all the lines (left), 50% of the mean liabilities are paid after two calendar years. For an individual line this is not necessarily the case – as illustrated for PL A (right) where it takes five years before 50% of the mean liabilities are paid out.

Correlations by calendar year

As with accident years, total calendar year correlations are calculated for the different aggregations including all lines as well as each component.
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**Risk capital allocation by calendar year**

Risk capital can be allocated between LOBs and across calendar years by the standard variance-covariance formula. That is, risk capital is allocated based on the LOB’s contribution to the total variability. It is easy to extract the necessary tables to allocate risk capital using other methods if desired.

Mathematically, percentage risk capital allocation to the ith line, \( L_i \), is:

\[
A_i = \frac{\sum_j C_{ij}}{\sum_{ij} C_{ij}}
\]

where \( C_{ij} \) is the covariance of \( L_i \) and \( L_j \).

The formula can be extended to include time (either calendar or accident period), by summation of the covariances across the relevant time period. Similarly, allocation across time periods for a single LOB can be readily considered by treating \( i,j \) as time indices rather than LOB indices.

**2.4. Aggregate reserve distributions for the three LOBs (eighteen components)**

The identified composite model predicts log-normal distributions, and their correlations, for each cell. Correlations are computed within and between segments, or, as in this case, cost components. Since there is no analytical closed distribution for the sum of correlated log-normals, simulations from the predicted correlated log-normals are conducted in order to obtain distributions of sums by accident year, calendar year, and totals for any segment and aggregation of segments. Quantiles (Percentiles), Value at-Risk (V@R) and Tail-Value-at-Risk (TV@R) can then be computed for any sum.

Simulations can be applied to a single cost component, LOB or the aggregate of all LOBs. All simulations can be saved to text files facilitating easy importation into other software – eg: DFA products.

An extract of the simulations by calendar year for the aggregate of all the lines is displayed.
The distribution of the total reserves, with the 75th percentile marked, is as follows:

Distributions can also be viewed by accident year or calendar year. The distributions by accident year above also mark the CRE on the outstanding distribution so an idea of IBNR can be ascertained.

The outstanding (reserve) distributions by calendar year show the means along with the boxplots for each calendar year where the whiskers of the boxplots indicate the 1st and 99th percentiles.
2.5. Risk capital allocation by LOB and cost component

The risk capital allocation by LOB shows Light Auto takes the largest allocation of risk capital. This is a feature of the volume of the LOB being the largest (see graph below) rather than the line being more volatile. In fact, Light Auto is the least volatile of the three lines.

Risk capital allocation by LOB is shown below (outer ring). The inner ring demonstrates the allocation to each cost component (unlabelled).
2.6. Quantiles (percentiles), V@Rs and T-V@Rs for the aggregate of the three LOBs

Quantiles (percentiles) are calculated from the simulations at specified levels and are available by either accident year or calendar year. The tails of the distributions can then be considered as a basis for risk capital requirements (Value-at-Risk or Tail-Value-at-Risk) in total, by accident year, or by calendar year. Since the quantile itself is a statistic, the uncertainty associated with the quantile is also calculated - whether by accident year or by calendar year.

The full V@R and T-V@R table displays quantiles from 1 through 99 with various levels above 99 (shown are entries from 96 through 99.9). For each entry, V@R and T-V@R are calculated for the simulations (sample), and the kernel, log-normal, and gamma distributions. The kernel is a smoothed version of the sample and is used in risk capital calculations. The log-normal distribution and gamma distributions are the corresponding distributions with the projected mean and standard deviation for the selected period (total, calendar year or accident year). Generally both the log-normal and gamma distributions are a poor fit to the upper tail of the sample distribution - the simulated correlated log-normal distributions have a much heavier tail.
2.7. Economic Balance Sheet and Solvency II and IFRS 4 metrics: SCR, TP, and Risk Margins

Solvency II for long-tail liabilities requires precise calibration of Solvency Capital Funds to a mandated stress level over the one-year horizon. Calibration must be done for individual Lines of Business (LOB) as well as aggregates of multiple Lines of Business, and possibly under required ring-fencing rules.

Actuaries will need access to accurate and precise distributional information about future cash flows and their modifications under stress. Only a unified approach to reserving which treats trends, volatility and correlations under a single distributional paradigm can achieve this result.

For further information regarding Insureware’s Solvency II one year risk horizon solution for long-tail liabilities please see “Solvency II - one-year and ultimate year risk horizons for long-tail liabilities”.

2.7.1. One-year risk horizon

The SCR for the one-year risk horizon is the distress Value-at-Risk for the first year plus the change (Δ) in technical provisions in the subsequent years (suitably discounted), conditional on the first year being in distress. This definition satisfies the directives and advice provided by CEIOPS (now known as EIOPA).

\[
\text{SCR} = \text{VaR}_{99.5\%}(1) + \Delta TP(2) + \Delta TP(3) + \ldots + \Delta TP(n);
\]

where \( n \) is the number of years until run-off.

The first year being in distress impacts the subsequent years - the effect is measured by the \( \Delta TP \). Including the adjustments changes the estimates of SCR and MVM for the first year. Recursion is not required if only the first year in distress is considered.

Calculated in this way, the SCR is adequate to restore the balance sheet to a fair value of liabilities at the end of a distressed first year so that the portfolio can then be transferred or sold to a reinsurer. That is, the economic balance sheet has sufficient SCR and TP to sustain a first year in distress and be restored to its fair value at the beginning of the second year. This formulation satisfies the Solvency II summary metrics.
The bulk of the paid losses should the company be in distress in the next calendar year arises from the light auto portfolio (the largest). The professional liability portfolio takes the next largest allocation of capital.

Allocation of total risk capital is shown above – note that percentages less than 3% are aggregated together into the ‘Other’ category for each LOB. In the case of Heavy Auto, all allocations are less than 3%. 
In the above table, the left-hand column is the simulation number in the distress band (the set of simulations reaching the distress threshold). It shows, for each simulation of a distress year, the contribution of each LOB to a distress event.

The lines marked in green (LA.A: Lost and LA.B) are much higher than their respective mean losses. The lines highlighted in blue are ‘in distress’ but do not contribute significantly to the total as their relative volume is low.

The secondary figures in the totals indicate the number of standard deviations above the mean the realised distress loss is (burgundy: >3 SDs, red > 2 SDs, orange > 1 SD; grey > mean). If the cell is empty then the loss is below the expected mean. Further details of Insureware’s Solvency II solution for long-tail liabilities is available in “Solvency II - one-year and ultimate year risk horizons for long-tail liabilities”.

2.7.2. Ultimate year risk horizon

The ultimate year risk horizon is typically calculated at a lower percentile (here 95%). The total risk capital at 95% is 202.2M. The MVM is calculated as well, but typically would not be required as the risk capital would be held internally.
The ultimate year risk horizon metrics (by calendar year: right) are closely related to the calendar year forecast summary (left). The calendar year liability stream standard deviations (and their correlations) drive the level of risk capital required to cover subsequent calendar years.

The economic balance sheet for the ultimate year risk horizon is shown below followed by the risk capital over calendar time.
2.8. Updating, monitoring, and reserve releases

When deciding on forecast scenarios, we take the approach of considering reasonable scenarios going forward – scenarios that are conservative in terms of future liability stream, but not unreasonably so. By comparing these reasonable scenarios to more optimistic scenarios, we can determine the amount of reserves that could be released (subject to management strategy) from the reserves held should the more optimistic scenario play out next year.

There are three cost components for which we create optimistic scenarios to illustrate the process. The original reasonable scenario is on the left, the more optimistic scenario on the right. Calendar trends to the left of the vertical green line are calendar year trends measured from the data; on the right of the green line are the calendar year trends assumed for the future.

All trend parameters going forward have a mean and uncertainty associated with them. The distribution used for the parameters is a normal distribution. For example, the future calendar trend of 24.17%±7.03% for PL D&O (bottom left) effectively implies that the parameter is a random draw from a normal distribution with mean 24.17% and standard deviation 7.03%. Scenarios going forward may vary parameter means, parameter uncertainty, or both.
By comparing the means of the two distributions, we see that up to 67M could be released from the reserves should the more optimistic scenario arise next year. The same strategies can be applied in subsequent years so any reserve releases can be amortised over time.

Further, we only consider the mean above, but we can immediately see that the volatility has also changed and is lower in the more optimistic scenario. Thus, risk capital can also be reduced if the more optimistic scenario arises. The corresponding reduction in risk capital can also be considered one year from now.
2.8.1 Risk capital release

If risk capital was considered at the 75th percentile, then, in addition to the difference in means (67M), up to 12M of risk capital can also be released. That is, the Value-at-Risk of 72M minus the Value-at-Risk of 60M. Equivalent calculations would be done if Tail-Value-at-Risk were used.

The implication of the above considerations is that risk capital management is an essential component of sensitivity analysis. Although a difference in means is one important (and influential) component of sensitivity analysis, risk capital levels are also of significant import and should not be overlooked. Above two scenarios were considered (a reasonable scenario versus a reserve release scenario), but in practice this can be extended to any number of forecast scenario assumptions.
2.9. Pricing future accident (or underwriting) years

In the MPTF (and PTF) modelling frameworks, the forecast scenario can extend to future accident (or underwriting) periods. Distributions are calculated for the aggregate of all the lines, each individual line, and each cost component. Pricing and allocation of risk capital charge can also be done by LOB or by cost component.

Total aggregate ultimate cost statistics for accident year 2012 for the three lines are as follows:

Above are the forecast means and standard deviations for the next accident year for the individual component Light Auto A (LA A). The black values are the fitted means, the red values are standard deviations of the correlated log-normal distributions. The burgundy values are correspond to the sum of the correlated log-normal distributions (outstanding column) and the standard deviations of the log-normal distributions by calendar period total (since there is only one cell to sum).

The total aggregate reserve distribution is comprised of the three lines as follows:
Modelling multiple long tail liability lines

The relative means and CVs are shown below for the next accident year (2012).

The entire distribution can be considered and the risk capital calculated for the aggregate outstanding distribution. Risk capital can be allocated by LOB, by accident year, or by calendar year (the latter being important for asset-liability matching).
Future risk capital allocation percentage (according to the variance-covariance formula) is shown above. Similarly, Light Auto (278M) above can be split into the three segments comprising this line:

or by cost component if this level of detail is required:

The summary statistics (means and standard deviations) are provided above for each cost component. As with the reserve distributions considered previously, simulations can be run at the cost component level in order to obtain a more complete picture of the future underwriting year distributions - whether in total, by accident year, or by calendar year.
2.10. Combined, reserve and underwriting risk

The section of the combined table highlighted in green comprises the projections for the reserve distribution. The future accident (underwriting) year is highlighted in blue. Projected distributions combining both the projected reserve and future underwriting year distributions are coloured in yellow. The totals in yellow include risk diversification between the reserve and future underwriting year loss distributions.

The future liability stream for the aggregate is reasonable compared to recent past payments. For instance, for 2009—2011 the payments are 379M, 370.7M, and 380.6M compared to the mean projected future calendar years 2012—2014 of 375.6M, 345.6M and 227.8M. Note the exposure for 2012 was assumed to be the same as 2011.

Risk capital is now considered at the 95th percentile and compared between combined, reserve, and underwriting risk. The selected percentile of 95% is arbitrary - any percentile can be used for comparison where the risk capital is positive.

Risk diversification credit gained for Prof. Liab. and Light Auto particularly is illustrated above. Although Heavy Auto takes a higher percentage (marginally) when examining the combined risk, the total capital is still less (Combined 6.8M vs 7M from Reserve 5.7M + Underwriting 1.3M).
The aggregate risk capital for various percentiles for the total reserve distribution is shown above. The 95% quantiles are highlighted and allocates the total of 199.2M to the three LOBs.

The aggregate risk capital for various percentiles for the total future underwriting year distribution is shown above. The 95% quantiles are highlighted and distributes the total risk capital of 81.4M to the three LOBs.

The combined risk capital for various percentiles of the total reserve and future underwriting year distribution is shown above. The 95% quantiles are highlighted and calculates the total risk capital for the combined distribution of 249.2M. The 249.2M for the combined capital is less than the sum of the two individual risk capital calculations (199.2M + 81.4M). This illustrates the effect of risk diversification when considering reserve and future underwriting risk as a joint problem. Risk capital is sub-additive.
3. **Company’s A and B: Credibility modelling**


Company A is considering purchasing Company B to extend its portfolio. Since the losses are written only between 1998 and 2001, only these data are made available from Company B.

3.1. Model displays

A model for Company B using just the data available for Company B is as follows:

It is immediately apparent that there is insufficient data to determine when the final development trend starts decaying. Further, it is impossible to detect a significant change in calendar year trends for 2000–2001, 2001+.
The model for company A, on the other hand, demonstrates clear changes in development year and calendar year trends as follows – note the final development trend decay.

We now develop a model for Company B’s data using Company A’s model.

The original model for Comp A is shown on the left. The model on the right for Comp B. The red bars are trends set to be in common with Comp A1 [a duplicate(*) of Comp A]. The final development trend decays are constrained to the parameters estimated for Comp A. The calendar year trend assumption is not significantly different from zero, but is not optimised to zero in the model due to the high volatility and the small data sample. Further, the calendar year trend in company B is not set to be the same calendar year trend as in Company A as the estimates of the parameters (A: 0.298+_0.0644 and B: 0.0544+_0.0614) are very different. Parameters are only credibility adjusted where it is reasonable to do so; the adjustments must be statistically sound.

(*) Duplication is necessary so the final projections can consist of Company A and the credibility adjusted Company B. In this way, company A’s estimates are independent of Company B’s credibility adjustments.
3.2. Aggregate forecast results

The Comp B outstanding reserve is the same magnitude as for Comp A. The reduction in volatility by adding this portfolio with Comp A is not immaterial – the projected losses for Comp B are substantially less volatile.
4. Company S: Losses and recoveries

Company S writes six lines of business: Private Motor, Commercial Motor, Professional Indemnity, Employers Liability, and Commercial Property. As with Company M, these lines of business are split into various components and, in the case of Private Motor and Commercial Motor, include recoveries.

Recoveries are modelled in the same fashion as loss components. However, unlike the loss components, recoveries are subtracted from the total forecasts. That is, all forecasts are net of recoveries.

From the identified composite model at the component level, aggregates at any upper layer can easily be created. A composite model is designed for the whole company with a complete view into any layer and cost component including the subtraction of recoveries.

4.1. Aggregate forecast results

The breakdown of the company’s portfolio into the five LOBs, net of recoveries, is shown below.

The percentage each line takes of the total reserve mean, along with the relative CVs for each line, are displayed in the graphs below. Note the CV of the aggregate reserve is only 3.11%.
The percentage each line takes of the total reserve mean, along with the relative CVs for each line, are displayed in the graphs below. Note the CV of the aggregate reserve is only 3.11%.

Forecast scenarios can be created for the aggregate of forecast combinations. Below are the multipliers to create each forecast scenario for the aggregate of all the lines (Left) from the LOBs and the multipliers to forecast the LOB PM (note the negative for Recoveries [Rec]).

The above tables show the allocation to each line and recoveries. While the means are additive, the standard deviation for the aggregate is calculated using the correlation matrix.
4.1.1. Summaries by accident year and calendar year

The breakdown by accident year and calendar year are shown above/below respectively. Calculations, including the one-year ahead conditional statistics, are available for the aggregate forecasts including those containing negative multipliers (i.e., recoveries).
4.2. Aggregate reserve distributions for the five LOBs (net of recoveries)

Simulations for the total reserve, net of recoveries, are calculated and the complete reserve distribution shown above. The value-at-risk at the 75th percentile is 3.4M €. All simulations can be exported to text files and imported into any other application of choice.

4.3. Risk capital allocation by LOB (net of recoveries)

The total risk capital percentage allocated to each LOB is displayed above where the variance-covariance formula was used to allocate capital. Recoveries are taken into consideration when allocating risk capital.
5. **A.M. Best Schedule P: Berkshire Hathaway, Swiss Re, and the Industry**

The A.M. Best Schedule P, NAIC Schedule P, or S&P SynThesis data can be imported into an ICRFS-Plus™ database.

The unique technological power of ICRFS-Plus™ combined with A.M. Best’s or NAIC Schedule P (USA) or S&P SynThesis (UK) data will give your company a strategic edge. Importing the data into an ICRFS-Plus™ database obtains all the data organisation, customisation, and modelling capabilities of ICRFS-Plus™. Gain a competitive advantage by comparing your company’s intrinsic risk characteristics and loss costs with those of your competitors.

For instance, the composition of companies can be compared (here BH and SR):

5.1. **Total reserve mean by LOB (BH vs SR)**

Company Berkshire Hathaway (BH) writes a large proportion of Primary Passenger Automobile (PPA), whereas Swiss Re (SR) writes a large proportion of Reinsurance portfolios (Re B and Re A consist of over 50% of the total reserve for SR).
5.2. CV (%) of reserve distribution by LOB (BH vs SR)

The CV of the aggregate reserve distribution is much higher for SR (17.2%) compared to BH (4.7%) – a feature of the large reinsurance portfolios.

5.3. Mean and risk capital by LOB (BH vs SR)
Company SR needs a significantly larger proportion of risk capital (relative to the mean) to reach the 95th percentile. The value-at-risk at the 95th percentile is almost the same for the two companies, but the total reserve mean for SR is one-third that of BH.

5.4. Calendar year trends in Commercial Auto Liability (CAL): BH vs SR vs Industry

Commercial Auto Liability (CAL) lines were selected from the companies BH and SR along with the total CAL Industry. The calendar year trends for these two companies for the CAL line are displayed below followed by the calendar year trends identified in the total CAL industry data.

The calendar year trends are different for each company’s CAL LOB and compared with the total CAL industry. The experience of each company is unique and an appropriate model which identifies the trends in the three directions along with the volatility around those trends is an absolute necessity for obtaining critical information about the business. Further, reasonable forecast scenario assumptions can only be made when information regarding past trends in the company’s data are quantified. It is clear from the above that trends in the industry are not a reliable source of information for trends in an individual company.
6. **Worker’s compensation segments: SAD and SAM**

Consider the following two segments of Worker’s Compensation written in California: SAD (left) and SAM (right). The red bars indicate common parameters between the segments. The calendar and development year parameters differ slightly, but the accident year parameters move synchronously with the result that the mean ultimates vary synchronously.

As discussed in “Understanding correlations and common drivers”, the similarity of the accident year mean ultimates does not imply volatility correlation. The mean ultimates move synchronously (left) and a graph of the mean ultimates of SAM versus the mean ultimates of SAD (right) shows an almost perfect linear relationship.

The linear relationship in mean ultimates is important when forecasting future underwriting (accident) years, but is not correlation in random effects (volatility). For instance, if the accident year level for one segment is expected to increase by 10% + 2%, then the other segment is also likely to increase by 10% + 2% in the same accident year.

The trends in the three directions must be quantified before measuring process (volatility) correlation. The ‘correlation’ between mean ultimates is then found best explained by similar trend adjustments in the model and forms the known effects. The process correlation is then correlation in randomness - the unknown effects.

Above, the residual displays with scatterplot (inset) for SAD and SAM are shown for a model which does not describe the accident year changes. The correlation (0.897) is very high, but it should be immediately apparent that this is not all correlation in randomness - there are distinct changes in level across the accident years (as indicated by the red arrows). These level changes, when quantified, comprise known effects and ensure the mean ultimates do move synchronously as they should.
In the model, with the accident year trends correctly fitted, the volatility correlation between the segments is related about 0.25. The result is that the mean ultimate losses (by accident year) move synchronously (and likely indicate common drivers), however the risk factors arising from volatility are not.

The summary tables by accident year for the two pieces with the correct volatility correlations are shown below.

The reserve distribution correlation is only 0.086! The reserve correlation is the correlation in the losses not explained by the means – and therefore is the critical measure when evaluating risk diversification. Models that do not capture the trends in the three directions in the data may indicate spurious correlations and erroneous conclusions. It is also important that the weighted standardised residuals of each model can be regarded as a random sample from a (normal) distribution. This way, the process (volatility) correlation can be measured correctly.

The common accident year drivers for the two segments are an important consideration for pricing future underwriting years. Both segments are expected to move in a similar fashion and this information must be included in any future forecast assumption - see “Understanding correlations and common drivers” for more details.