Link ratios, Mack, Murphy, Over-Dispersed Poisson and the bootstrap technique

Insureware
Software Solutions and eConsulting for P&C Insurance

Contents

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1. Introduction: Challenging the link ratio methods

Methods based on link ratios are among the most widespread techniques for obtaining best estimates of reserves. There are three primary reasons for this. First, they are mathematically simple to calculate in a spread-sheet and do not presuppose any statistical analysis. Second, with the formulation of volume weighted average (chain ladder) link ratios as regression equations, the methods 'became stochastic' and were able to accommodate the industry's need for 'ranges'. Third, further enhancements by way of the bootstrap technique have provided link ratios with a response to the demands of modern risk management, such as those emerging from Solvency II.

In spite of their popularity, link ratio methods are also known to have fundamental drawbacks and are being superseded by new scientific developments. In this brochure we challenge the pervasive use of link ratios as a method of calculating reserves and present an up to date alternative.

In particular, we demonstrate that link ratio methods:

- are vulnerable to biases which can produce wildly inaccurate forecasts because they fail to provide sufficient descriptors of data - especially of calendar year trends and volatility;
- lack the methodological flexibility to respond to problems which can be detected by simple diagnostic tests, and hence these tests are often overlooked or ignored;
- rely on hidden assumptions about the nature of the data which are often unmet; and
- cannot be corrected by 'stochastic' formulation or the use of the bootstrap technique – these ancillary methods do not compensate for errors in model parameterisation at the base level.

To address these issues, a statistical modelling framework which is transparent in its assumptions, adaptive to the key trends in the data and responsive to internal diagnostics, is presented. This framework not only meets and resolves the modelling problems listed above, but is also surprisingly simple to use.

1.1. What are link ratios really?

A link ratio y/x is the ratio between two successive cumulative numbers x and y at successive development periods. For a given pair of contiguous development periods within the triangle, there is one ratio for each accident period.

The ratio y/x is the slope of the line between the number pairs (0,0) and (x,y). It is a trend. A weighted average link ratio is therefore an estimate of an average trend.

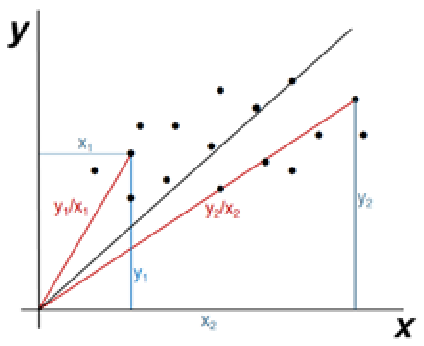
The true relationship between two successive development periods is considered to be captured by a weighted average of these link ratios. These weighted averages are thus calculated in order to project a best estimate of cumulative numbers at future times.

Note when projecting the next cumulative using a link ratio, it is the difference between the two cumulatives that is actually being projected; this difference is known as the incremental. See Section 1.3.

Can they be viewed graphically?

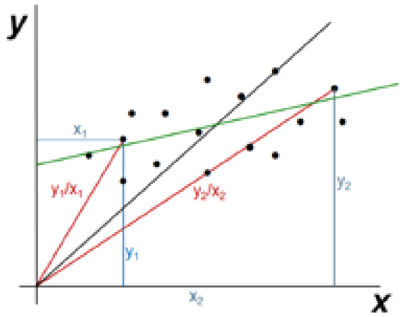
Setting successive cumulatives as the axes on a graph the link ratios can be viewed as the slopes of lines. This simple display of the cumulative data versus the previous cumulative allows an instant visual assessment of how well an average link ratio can describe the (historical) trend between the cumulatives.

The link ratios are displayed below for a set of cumulatives (y) and the previous cumulatives (x). Each point represents a distinct accident period. The chart shows two link ratios in red and marks their slopes. An average of all the link ratios is the trend illustrated by the diagonal black line.



The graphical representation above naturally leads to a regression formulation of link ratios as discussed by Mack(1993). The regression equation in Mack(1993) excludes an intercept so the link ratios go through the origin. Furthermore, in the Mack(1993) regression equation the variance of the next cumulative (y) about the average trend (ratio) is proportional to the current cumulative (x). The weighted least squares estimator of the average link ratio is then the volume weighted average (equivalently the chain ladder ratio).

The black average line through the origin is clearly not the best line to represent the observations. A better line is the green line (below) which is not constrained through the origin. Regression formulations of this type of model are described in Murphy(1994).



1.2. The Mack method - a regression formulation of chain ladder link ratios

The Mack method is a regression formulation of volume weighted average link ratios.

The regression equation for the Mack method is:

$$y = bx + \varepsilon \ \ : \ \ V(\varepsilon) = \sigma^2 x^\delta$$

where:

- y 's are the next cumulatives;
- x 's are the current cumulatives;
- b is the weighted average link ratio; and
- $\delta = 1$.

Here, and in all regression equations, ε refers to the 'error', the difference between the observation, y, and its mean value bx.

The variance of y about the mean value bx is given by σ^2x.

The best weighted least squares estimator $\hat{b}$ of b where the weight is proportional to the inverse of the variance 1/x, is the volume weighted average link ratio, equivalently, the chain ladder ratio.

The fitted value $\hat{\epsilon} = y - \hat{b}x$ of ε is called the residual.

A necessary (but not sufficient) condition for the model to be appropriate for the data is that the weighted standardised residuals are random versus development period, accident period, calendar period and fitted values.

Mack (1993) derived standard deviations of reserves by accident year and total for the above regression equation only in the case of $\delta = 1$.

The cases $\delta = 0$ and $\delta = 2$ were studied by Murphy (1994). When $\delta = 2$, the best weighted least squares link ratio is the arithmetic average, and when $\delta = 0$, the best average is volume weighted squared.

1.3. The Murphy method

The Murphy method is more general than the Mack method. Murphy (1994) also calculated the two cases $\delta = 0$ and $\delta = 2$ and more importantly introduced an intercept a.

$$y = a + bx + \varepsilon \ \ : \ \ V(\varepsilon) = \sigma^2 x$$

or, equivalently, according to Venter(1998)

$$y - x = a + (b-1)x + \varepsilon \ \ : \ \ V(\varepsilon) = \sigma^2 x^\delta$$

where:

- y's are the next cumulatives;
- x's are the previous cumulatives;
- y - x is the next incremental;
- a is the intercept term;
- b is the average link ratio (given the intercept, a); and
- $\delta = 0$, 1, or 2.

The regression formulation of the link ratio methods above are an excellent contribution to actuarial literature because this formulation also allows the methods to be statistically tested. The assumptions made by the methods can be detailed and verified.

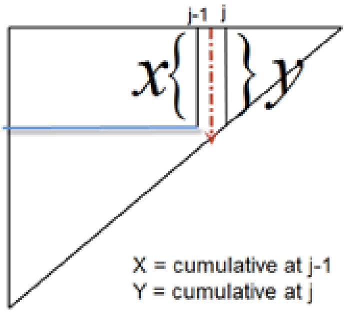
The incremental formulation by Venter (1998) is particularly important. Link ratios predict the next incremental conditional on the previous cumulative. Both the cumulative versus cumulative regression and incremental versus cumulative regression are correct (and equivalent). However, conceptually the focus should be on the latter.

In the incremental versus cumulative formulation one tests the significance of b - 1 in the presence of an intercept. The null hypothesis H_0 : b - 1 = 0 is equivalent to the incrementals y - x not being correlated to the previous x. That is, the link ratios having no predictive power.

1.4. The Extended Link Ratio Family (ELRF) modelling framework

There is one further component of interest that can be added to the formulation, for which δ can take values 0, 1 or 2.

We can include a constant trend in the incrementals down the accident years for each development year, (red arrow).



The models in the Extended Link Ratio Family (ELRF) modelling framework can be written;

$$p = a_0 + a_1 w + (b-1)x + \varepsilon : V(\varepsilon) = \sigma^2 x^\delta$$

where:

- $p = y - x$; as per Venter(1998);
- a_0 is the intercept term:
- a_1 is the constant trend down the accident years:
- b is the average link ratio (given the intercept, a_0, and the constant accident year trend, a_1):
- w is the accident year:
- x's are the previous cumulatives: and
- $\delta = 0, 1$, or 2.

In this framework the most optimal combination of intercepts, trends (down the accident period), ratios, and variance assumptions can be found.

1.5. Four assumptions made by link ratio methods and how to verify them

The regression formulation of link ratios (Mack, Murphy, and other extensions), provide the framework for verifying the assumptions made by the link ratio techniques. We have found these assumptions are rarely satisfied - unless the data is simulated using ratios.

If using link ratio based methods, it is critical that actuaries verify the corresponding assumptions apply for the company's data.

The regression formulations are distribution-free, but are certainly not assumption free. For linear regression to be optimal a number of basic assumptions must be satisfied (or at least approximately satisfied).

1. The underlying relationship between the y's and the x's is linear and for link ratios goes through (0,0); equivalently independent of scale;
2. The residuals are random about zero:
    2.1. versus time (development, accident, or calendar);
    2.2. versus fitted values;
3. The variance of the residuals is proportional to x (when using the volume weighted average (Mack) methods), x^2 (arithmetic average), or constant;
4. If a calendar year trend exists, it is constant.

Link ratio methods capture an average calendar year trend, but there are no descriptors of it!

ICRFS-PLUS™ and ICRFS-ELRF™ provide displays for testing these assumptions. The significance of an intercept can be tested using the regression and diagnostically by viewing the Y versus X plot, whereas the significance of the ratio minus one (b - 1) can be tested using the regression and diagnostically by viewing the Y - X versus X plot. Residual plots versus development period, accident period and calendar period are also used to assess model specification error.

Real data may conform to some, but almost never to all the four assumptions. Violation of any of the assumptions means that little confidence can be assigned to forecast numbers from the associated link ratio methods.

What about the bootstrap technique?

The technique is primarily used for generating distributions of loss reserves. It is inherently linked to a model. The technique generates bootstrap triangles by sampling the residuals (with replacement) and adding them to the fitted values. Estimating the reserves using the link ratio method applied to the bootstrap triangles generates loss reserve distributions.

Another application of the bootstrap technique is in the context of assessing model specification error - discussed below.

1.6 Model specification error: minimising model risk

If a model is misspecified for the data, then all results based on the model have nothing to do with the data. For example, probabilities derived from a model for tossing a fair coin 100 times have no relevance to the process of spinning a symmetric roulette wheel numbered 0, 1, 2, ... 120 - and vice versa.

How do we determine whether a statistical probabilistic model is misspecified or specified correctly?

A good model replicates the volatility in the real data. That is, loss development arrays simulated from a good model are indistinguishable from the real data in respect of salient statistical features. As the discrepancies between simulations and the real data increase, the model specification error increases.

Given no assumptions are made about the distribution of the error term, ε, in the link ratio regression formulations; probabilistic distributions cannot be used to generate simulations. In this situation, the best we can do is to use the bootstrap technique to create bootstrap samples which are compared to the real data. If the bootstrap samples do not share the same features as the real data, the model has been specified incorrectly and any inferences from the model or the bootstrap samples are meaningless.

If the link ratio methods are deficient, then the assumption tests will fail. Simulations will be distinguishable and the bootstrap technique will confirm this evidence.

1.7. Judgment - does it help? The “black-box” problem

Judgment refers to the technique replacing link ratios calculated according to a pre-defined method with others chosen on an ad hoc basis.

Can judgment produce an accurate forecast? Yes, if you’re lucky!

Can judgment produce a reliable forecast? No!

Judgment is basically guesswork, and while it may perform better than rigid adherence to a ratio method that is known to be inaccurate the quality of the results it produces rests entirely on luck.

The same can be said of forecasting techniques which rely on averaging the results from a number of methods which individually lack conviction and where there is an absence of diagnostics which can identify when all of the polled methods share the same bias.

Calendar trends in the data are a common feature. They typically produce systematically biased answers across the entire range of ratio methods.

There is no regression formulation of judgment - which causes issue with variability. Further, judgement is very difficult in the presence of high volatility.

The problem with judgment as a technique highlights the more general problem of so-called “black-box” methods where the precise logic leading to a particular forecast is hidden. Actuarial forecasts need to be fully auditable and hence a further desideratum of models is that they be transparent.

In summary, a model should tell a clear story about the past data and about how this story has been extended to produce a consistent forecast of future results.

1.8. Three reasons link ratio methods are not appropriate modelling tools for long-tail liabilities

- The assumptions underpinning the link ratio techniques (section 1.5) are rarely satisfied by real data - this leads to model specification error;
- Even if the assumptions are satisfied, the model produces no insight into the forces driving the data – it does not make the data intelligible;
- Actuarial judgement or additional treatments cannot compensate for model deficiencies:
    - The effect of calendar year trend changes on the average link ratios is unknown;
    - The efficiency of the link ratio methods (and any adjustments for judgement) is unquantifiable.
    - Bootstrap simulations cannot compensate for an imperfectly specified model – the resulting bootstrap distribution reflects the model misfit rather than identifying features intrinsic to the data.

These deficiencies have been known in the industry for over a decade, (Barnett and Zehnwirth (2000)) and yet methods based on link ratios are still commonly used to estimate loss reserves.

When using link ratio based methods, it is critical that actuaries verify that the corresponding assumptions apply for the data.

1.9. Outline of remaining sections

Section 2 comprises several case studies demonstrating link ratios and their shortcomings.

The Mack(1993) data demonstrates over fitting of large observations and under fitting of small observations; link ratios become statistically indistinguishable from unity in the presence of an intercept.

Company ABC has changing calendar year trends; no link ratio method can describe the calendar year trends resulting in under-projection.

Company LR High also has changing calendar year trends, but where the trends have been constant for the most recent seven calendar years. All link ratio methods calculated on all the calendar years overstate the reserves significantly (the Mack method by a factor of two!).

Section 3 shows that the chain-ladder (or volume weighted average) method, the most popular of the link-ratio methods encodes a key symmetry (interchangeability of development and accident dimensions) which is not present in the state of affairs that is being modelled. The symmetry exists in respect of the forecast mean, but not the forecast variance, leading to two entirely different variance estimates.

Section 4 describes the bootstrap technique as it should apply to link ratio methods. We outline the correct application of the bootstrap and show the technique, as a diagnostic, identifies the same issues revealed in the case studies (Section 2).

Section 5 shows that the Mack method can lead to spurious correlations appearing between lines of business as the method fails to describe calendar year trends. The correlation measured is shown to appear due to the commonly missed calendar year trends rather than being true volatility correlation.

Section 6 demonstrates two examples where, if the Mack method was applied, significant adverse development (losses greatly exceeded expectations) would (and did) occur. In this section we show that if a modelling framework which incorporated calendar year trends was used (like the Probabilistic Trend Family (PTF) modelling framework in ICRFS-PLUS™), disaster have been avoided since the optimal PTF model identifies the calendar year trends and projects them into the future. The subsequent losses over the next three years fall along the projected trend lines – the adverse development arose due to the poor model choice, not due to genuine ‘unexpected’ losses.

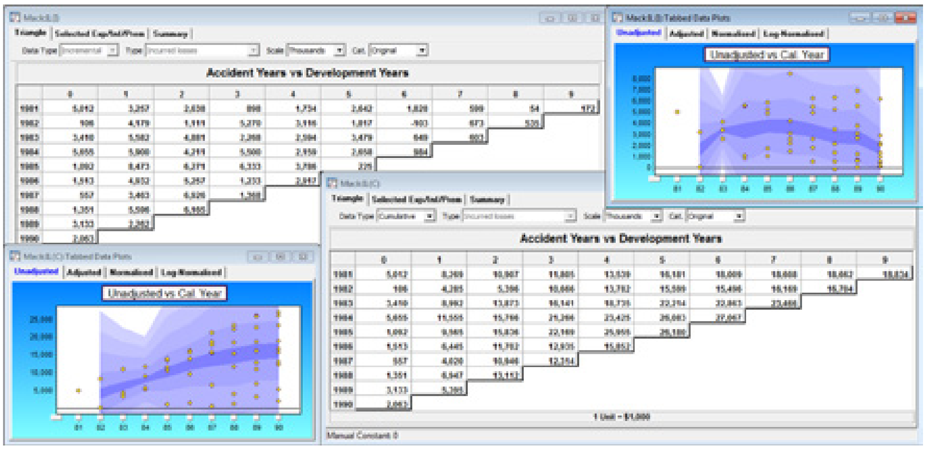
2. Case study: Extracting information from link ratios

2.1. Mack data: Link ratios including the Mack method have no predictive power

We consider the data analysed in Mack (1994) as originally published by the Reinsurance Association of America (RAA) in the 1991 Historical Loss Development Study.

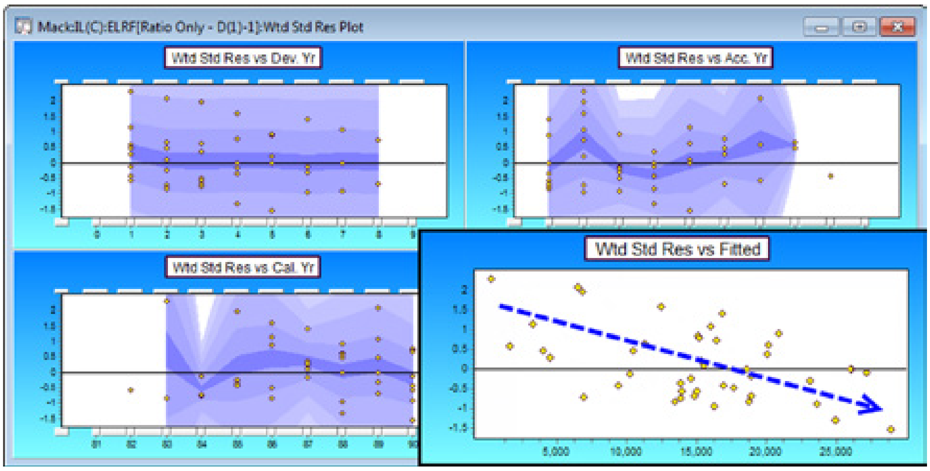
Link ratios have no predictive power; incremental incurred are not correlated to the previous period cumulatives. The best model in the ELRF modelling framework comprises only an intercept save development period 2 that includes a constant trend in the incrementals down the accident year.

The data are illustrated below on an incremental scale (top) and a cumulative scale (bottom). Which form provides more insight into the trends in the data?



We begin by fitting the Mack method and examining the residuals.

The residuals versus the three time directions (development, accident, and calendar), appear fairly random. However, the residuals versus fitted (enlarged lower left) indicates a clear pattern: small values are underestimated and large values are overestimated.



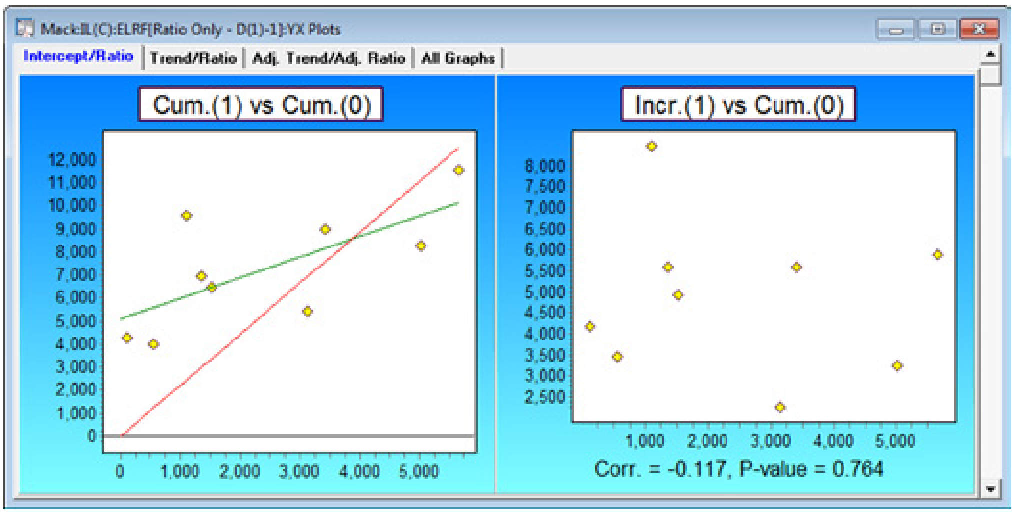
The average calendar year trend (seen in the cumulative series) has been quantified, but the method has not provided an estimate of this trend.

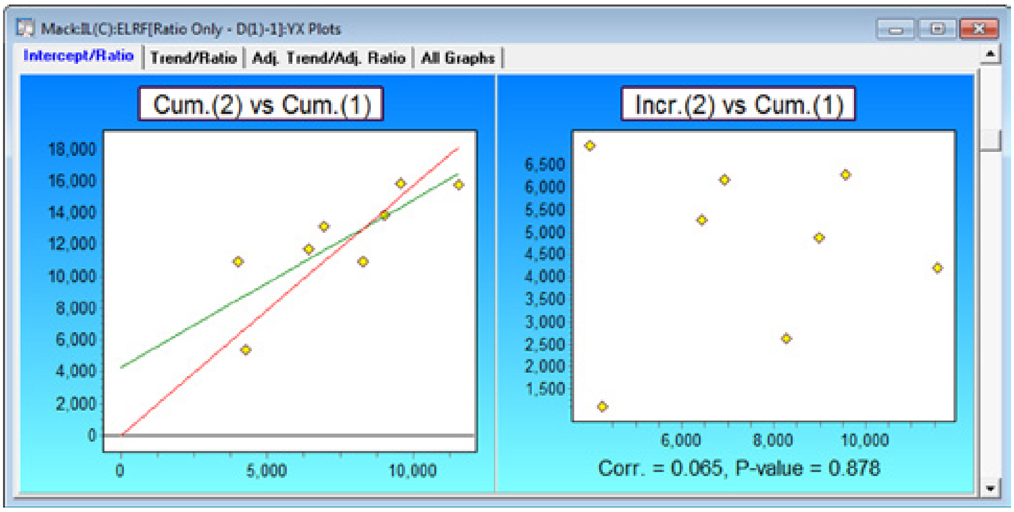
Predictive power

The first step to determine whether link ratios have predictive power is to first determine whether an intercept is required in the regression model. The leftmost graphs below show the current cumulatives versus the previous cumulatives. The red link in each shows the traditional link ratio (line is constrained to go through the origin). The green line illustrates the ratio in the presence of an intercept. The fit of the lines can be compared graphically. In both cases, the green line clearly is closer to the observations than the red line.

If the green line gives a better fit, then we can then test to see whether the ratio, in the presence of the intercept, has predictive power.

The incremental losses in development year one are not correlated to the previous cumulative losses **in the presence of an intercept**. Similarly, the incremental losses in year two are not correlated to the cumulative losses as at development period one in the presence of an intercept. That is, we test whether the ratio - 1 is equal to zero. If the ratio - 1 is statistically zero, then the ratio has no predictive power.





The Y|X plots illustrate that the intercepts (the average losses in a column) are more important than the link ratios.

Extending Link Ratio Family model parameters

Volume weighted average link ratios only (Mack method)

MackIL(C):ELRF[Ratio Only - D(1)-1]:Regression Table

Mack method parameter estimates

Development Period	Intercept Est.	Intercept S.E.	Intercept P-Value	Trend Est.	Trend S.E.	Trend P-Value	Ratio Est.	Ratio Ratio-1	Ratio S.E.	Ratio P-Value
0~1	****	****	****	****	****	****	2.99936	1.99936	1.13020	0.11486
1~2	****	****	****	****	****	****	1.62352	0.62352	0.13584	0.00251
2~3	****	****	****	****	****	****	1.27089	0.27089	0.09050	0.02422
3~4	****	****	****	****	****	****	1.17167	0.17167	0.02539	0.00107
4~5	****	****	****	****	****	****	1.11338	0.11338	0.03538	0.03274
5~6	****	****	****	****	****	****	1.04193	0.04193	0.02258	0.16025
6~7	****	****	****	****	****	[illegible]	1.03326	0.03326	0.00488	0.02087
7~8	****	****	****	****	****	****	1.01694	0.01694	0.01506	0.46262
8~9	****	****	****	****	****	****	1.00922	0.00922	****	****
To Ultimate							1.00000	0.00000	0.00000	****

Delta = 1, AIC = 791.8
If the test is to be conducted at an overall 5% level, a parameter
would be regarded as insignificant if the corresponding P-Value is greater than 0.005683

Add intercepts (Murphy method)

MackIL(C):ELRF[Intercept and Ratio - D(1)-1]:Regression Table

Murphy method parameter estimates

Development Period	Intercept Est.	Intercept S.E.	Intercept P-Value	Trend Est.	Trend S.E.	Trend P-Value	Ratio Est.	Ratio Ratio-1	Ratio S.E.	Ratio P-Value
0~1	4,329.20580	516.30820	0.00007	[illegible]	****	****	1.21445	0.21445	0.42131	0.62640
1~2	4,159.69012	2,531.37466	0.15144	****	****	****	1.06962	0.06962	0.35842	0.85240
2~3	4,235.91788	2,814.51777	0.19266	****	****	****	0.91968	-0.08032	0.24743	0.75860
3~4	2,188.78926	1,133.10544	0.12557	****	****	****	1.03341	0.03341	0.07443	0.67679
4~5	3,562.27353	2,031.40955	0.17778	****	****	****	0.92675	-0.07325	0.11023	0.55389
5~6	589.27571	2,510.34558	0.83625	****	****	****	1.01250	0.01250	0.12833	0.93129
6~7	792.28254	148.92849	0.11829	****	****	****	0.99110	-0.00890	0.00803	0.46713
7~8	****	****	****	****	****	****	1.01694	0.01694	0.01506	0.46262
8~9	****	****	****	****	****	****	1.00922	0.00922	****	****
To Ultimate							1.00000	0.00000	0.00000	****

Delta = 1, AIC = 760.8
If the test is to be conducted at an overall 5% level, a parameter
would be regarded as insignificant if the corresponding P-Value is greater than 0.003201

In the presence of the intercepts, the link ratios become statistically insignificant (ratio – 1 = 0). Note the p-values of the ratios in the presence of an intercept now all indicate insignificance compared to the Mack method where most are statistically significant.

Optimisation can be performed by removing the parameters in each regression with the least significant t-ratio.

Optimise by removing parameters with the least significant t-ratio

MackIL(C):ELRF[Intercept and Ratio - D(1)?1]:Regression Table

Murphy method parameter estimates after optimisation

Development Period	Intercept Est.	Intercept S.E.	Intercept P-Value	Trend Est.	Trend S.E.	Trend P-Value	Ratio Est.	Ratio Ratio-1	Ratio S.E.	Ratio P-Value
0~1	4,461.99015	424.42206	0.00001	****	****	****	1.00000	0.00000	0.00000	0.00000
1~2	4,622.08480	799.18131	0.00067	****	****	****	1.00000	0.00000	0.00000	0.00000
2~3	3,374.17536	862.95694	0.00789	****	****	****	1.00000	0.00000	0.00000	0.00000
3~4	2,677.86913	284.92869	0.00023	****	****	****	1.00000	0.00000	0.00000	0.00000
4~5	2,259.01165	490.69263	0.01000	****	****	****	1.00000	0.00000	0.00000	0.00000
5~6	828.19349	436.73199	0.15419	****	****	****	1.00000	0.00000	0.00000	0.00000
6~7	629.22222	24.98294	0.00157	****	****	****	1.00000	0.00000	0.00000	0.00000
7~8	****	****	****	****	****	****	1.01694	0.01694	0.01506	0.46262
8~9	****	****	****	****	****	****	1.00922	0.00922	****	****
To Ultimate							1.00000	0.00000	0.00000	****

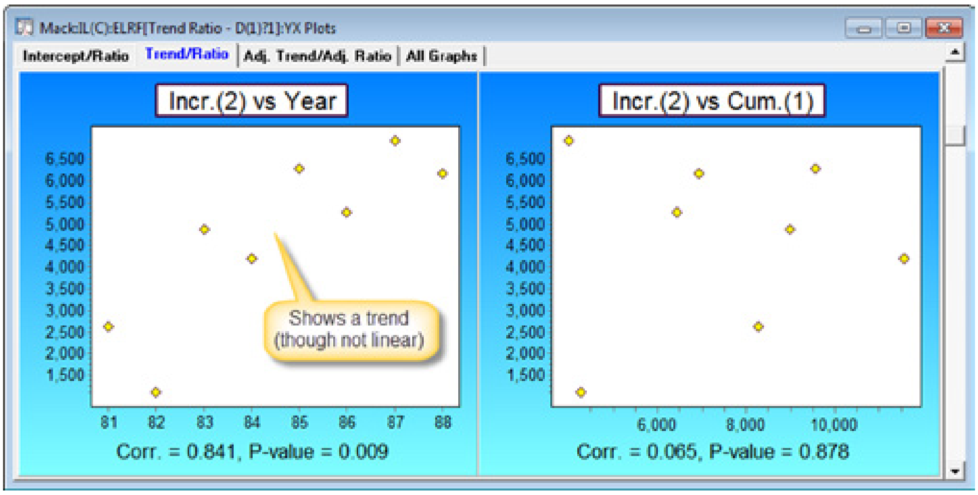
Delta = 1, AIC = 750.8
If the test is to be conducted at an overall 5% level, a parameter
would be regarded as insignificant if the corresponding P-Value is greater than 0.005683

The final, most optimal model, has removed the link ratios. The optimisation found the intercepts to be more useful for prediction than the ratios.

Link ratios are eliminated from the model; intercepts are retained!

In the optimised model, the best estimates are obtained by taking the average of the incremental losses in each column.

Are there any trends down the accident years? We find at development period 1~2 evidence of a trend – note link ratios (right) are not correlated to the previous cumulative despite this trend.



After fitting the model with intercepts, trends, and link ratios, the model is optimised – again parameters with the least significant t-ratios are removed first:

MackIL(C):ELRF[Trend Ratio - D(1)?1]:Regression Table

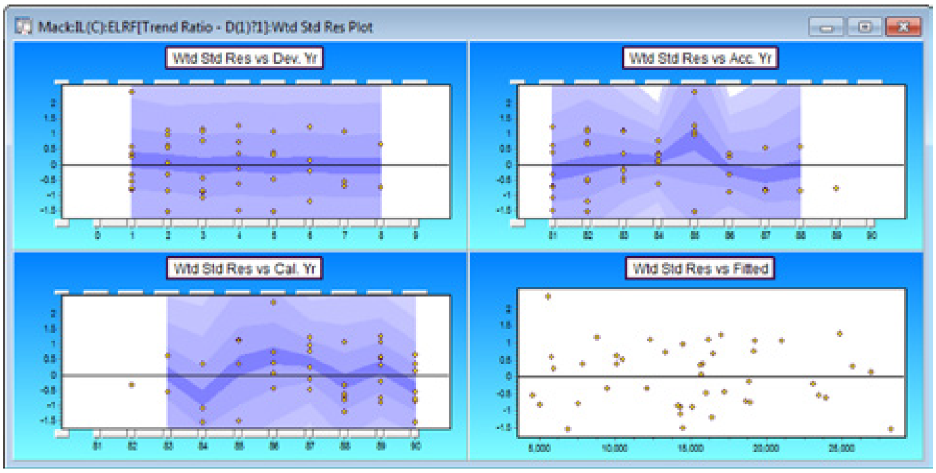
Intercept, trend, and link ratio parameter estimates after optimisation

Development	Intercept			Trend			Ratio			
Period	Est.	S.E.	P-Value	Est.	S.E.	P-Value	Est.	Ratio-1	S.E.	P-Value
0~1	4,461.99015	424.42206	0.00001	****	****	****	1.00000	0.00000	0.00000	0.00000
1~2	1,795.64670	772.46197	0.05908	772.03751	176.70388	0.00472	1.00000	0.00000	0.00000	0.00000
2~3	3,374.17536	862.95694	0.00789	****	****	****	1.00000	0.00000	0.00000	0.00000
3~4	2,677.86913	284.92869	0.00023	****	****	****	1.00000	0.00000	0.00000	0.00000
4~5	2,259.01165	490.69263	0.01000	****	****	****	1.00000	0.00000	0.00000	0.00000
5~6	828.19349	436.73199	0.15419	****	****	****	1.00000	0.00000	0.00000	0.00000
6~7	629.22222	24.98294	0.00157	****	****	****	1.00000	0.00000	0.00000	0.00000
7~8	****	****	****	****	****	****	1.01694	0.01694	0.01506	0.46262
8~9	****	****	****	****	****	****	1.00922	0.00922	****	****
To Ultimate							1.00000	0.00000	0.00000	****

Delta = 1, AIC = 741.3
If the test is to be conducted at an overall 5% level, a parameter would be regarded as insignificant if the corresponding P-Value is greater than 0.005116

Link ratios are removed and a trend has been added down the accident years for development periods 1~2.

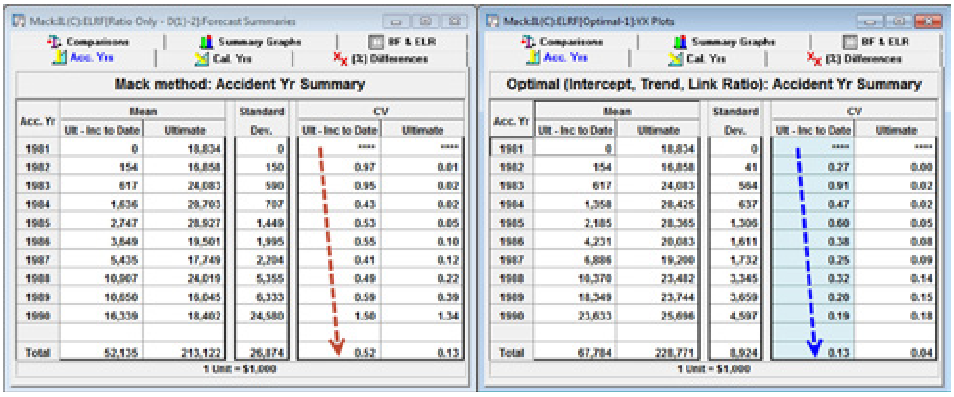
The residuals for the most optimal model are shown below.



While the residuals show a significant improvement on the residuals obtained from the Mack method, the method has not provided any insight into the data. All the only conclusion that can be drawn is that link ratios do not add any information to the modelling over the average level by development period or trends down the accident years.

Impact on the forecast

What impact does a poor model have on the forecasts?



Mack method: Accident Yr Summary

Acc. Yr	Mean		Standard	CV	
	Ult - Inc to Date	Ultimate	Dev.	Ult - Inc to Date	Ultimate
1981	0	18,834	0	****	****
1982	154	16,858	150	0.97	0.01
1983	617	24,083	590	0.95	0.02
1984	1,636	28,703	707	0.43	0.02
1985	2,747	28,927	1,449	0.53	0.05
1986	3,649	19,501	1,995	0.55	0.10
1987	5,435	17,749	2,204	0.41	0.12
1988	10,907	24,019	5,355	0.49	0.22
1989	10,650	16,045	6,333	0.59	0.39
1990	16,339	18,402	24,580	1.50	1.34
Total	52,135	213,122	26,874	0.52	0.13

1 Unit = $1,000

Optimal (Intercept, Trend, Link Ratio): Accident Yr Summary

Acc. Yr	Mean		Standard	CV	
	Ult - Inc to Date	Ultimate	Dev.	Ult - Inc to Date	Ultimate
1981	0	18,834	0	****	****
1982	154	16,858	41	0.27	0.00
1983	617	24,083	564	0.91	0.02
1984	1,358	28,425	637	0.47	0.02
1985	2,185	28,365	1,306	0.60	0.05
1986	4,231	20,083	1,611	0.38	0.08
1987	6,886	19,200	1,732	0.25	0.09
1988	10,370	23,482	3,345	0.32	0.14
1989	18,349	23,744	3,659	0.20	0.15
1990	23,633	25,696	4,597	0.19	0.18
Total	67,784	228,771	8,924	0.13	0.04

1 Unit = $1,000

The most optimal model (right) produces a total mean of 67.8M; 30% higher than the Mack method mean!

Notice the behaviour of the CVs by accident year (and total). The Mack method CVs do not decrease (and actually increase over the last four accident years). In contrast, the CVs by accident year for the optimal intercept, trend, and link ratio model generally decrease down the accident years. The more cells forecasted, the lower the CV should be. The Mack method CVs do not make mathematical sense - further evidence that the Mack method is not appropriate for this data.

Summary

Link ratio methods were found to be unsuitable for the Mack data.

- Link ratios do not provide any information regarding subsequent losses;
- Diagnostics reveal intercepts are required in the model;
- When intercepts are included:
  - o Link ratios are redundant – optimisation removes them.
- The most optimal method in this framework has not offered insight into trends in the data.

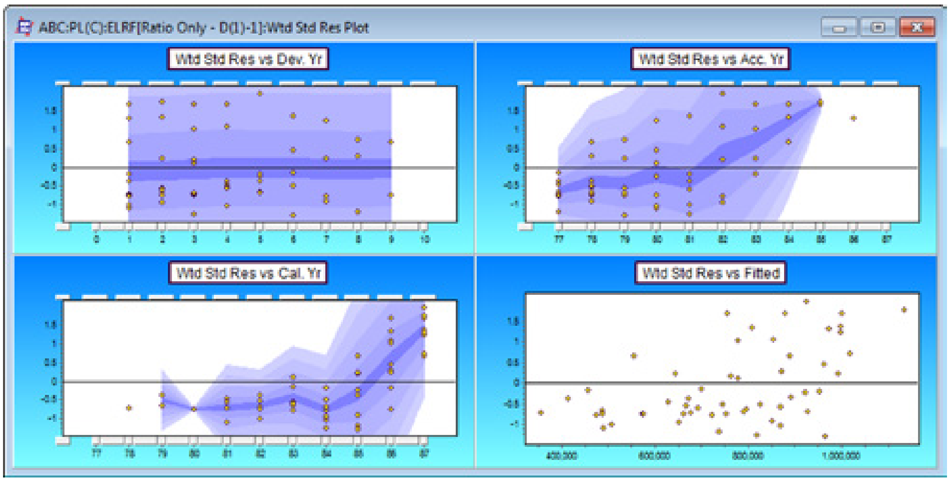
2.2. Company ABC: Link ratio techniques cannot handle changing calendar year trends

Data from a Workers Compensation portfolio is considered in which the residuals for all link-ratio-based method show a clear calendar trend change. Net inflation (social plus economic) increases to a high level in the last two calendar years.

Techniques based on these methods are unable to parameterise this change with the result that their projections are too low.

Mack method: **Residuals**

The ABC data was previously discussed by Barnett and Zehnwirth (2000). The Mack method is typical of all ratio-based regression methods in that its residuals show a marked increase in calendar trend in the three years after 1984.



Since the Mack method only fits an (unquantified) average calendar year trend, all that can be determined from the residuals is that the method underfits the most recent calendar year trends (and the larger numbers) and overfits the earlier calendar years (and the smaller numbers).

It is convenient to interpret the residuals as showing the trends in the data minus the trends in the method, hence what we see in the calendar direction residuals in the lower left is that in the model-estimated calendar trend falls far behind the data trend in later calendar years.

The net result is that the method projects an estimate of the total reserve mean which is most likely to be too low.

Mack method: **Forecast Table**

The misfit of model to data can also be seen by examining the forecast table. Historic data appears above the main diagonal with observed numbers in blue and fitted in black.

ABC:PL(C):ELRF[Ratio Only - D(1)-1]:Forecast

Incremental | Cumulative

Accident Period vs Development Period (Incremental Forecast)

	Cal.Per.Total	0	1	2	3	4	5	6	7	8	9	10
1977	153,638	153,638	200,221	143,992	96,028	64,815	46,277	31,901	24,037	18,721	14,840	12,200
	153,638	153,638	188,412	134,534	87,456	60,348	42,404	31,238	21,252	16,622	14,440	12,200
1978	378,757	178,536	232,668	170,471	113,609	76,822	54,844	37,699	28,359	22,081	17,564	14,455
	366,948	178,536	226,412	158,894	104,686	71,448	47,990	35,576	24,818	22,662	18,000	194
1979	586,833	210,172	273,896	197,578	132,527	89,723	64,050	44,029	33,016	25,833	20,567	16,918
	571,118	210,172	259,168	188,388	123,074	83,380	56,086	38,496	33,768	27,400	841	201
1980	751,843	211,448	275,559	195,721	130,627	89,555	63,617	43,947	33,201	26,083	20,734	17,055
	716,966	211,448	253,482	183,370	131,040	78,994	60,232	45,568	38,000	2,427	824	197
1981	871,371	219,810	286,457	204,639	137,168	92,028	65,848	45,508	34,511	26,983	21,449	17,643
	826,714	219,810	266,304	194,650	120,098	87,582	62,750	51,000	4,471	2,442	837	210
1982	943,458	205,654	268,009	192,972	128,130	87,957	63,904	45,193	34,085	26,650	21,184	17,425
	892,254	205,654	252,746	177,506	129,522	96,786	82,400	4,190	4,168	2,291	797	206
1983	977,480	197,716	257,664	190,751	130,521	90,792	66,386	46,029	34,715	27,143	21,576	17,748
	938,760	197,716	255,408	194,648	142,328	105,600	10,809	4,269	4,235	2,340	844	288
1984	1,042,930	239,784	312,487	239,542	168,009	117,696	84,635	58,683	44,258	34,604	27,507	22,626
	984,784	239,784	329,242	264,802	190,400	11,887	13,136	5,326	5,188	3,892	1,080	420
1985	1,204,426	326,304	425,240	335,953	236,440	162,013	116,504	80,779	60,923	47,634	37,865	31,146
	1,207,466	326,304	471,744	375,400	19,788	15,754	17,337	6,979	6,912	3,899	1,536	703
1986	1,483,769	420,778	548,358	425,674	289,514	198,380	142,655	98,911	74,599	58,326	46,364	38,138
	1,575,626	420,778	590,400	30,301	24,272	19,183	20,849	8,576	8,407	4,831	2,087	1,159
1987	1,837,524	496,200	646,648	481,104	327,213	224,212	161,231	111,791	84,313	65,921	52,401	43,104
	1,987,000	496,200	39,107	36,938	28,780	22,339	23,494	10,157	9,688	5,788	2,918	1,943
	Total Fitted/Paid		1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
Cal. Yr Totals	10,232,007		1,633,654	1,162,014	800,648	557,874	394,671	279,278	203,130	143,431	90,539	43,104
	10,229,194		####	####	####	####	####	####	####	####	####	####

1 Unit = $1

The calendar years highlighted in yellow demonstrate over-fitting: the fitted model means (black numbers), are higher than the observed values (blue numbers). The last two calendar years highlighted in red are substantially under-fitted: the model means have not increased in line with the calendar year trends.

The deployment of an average calendar year trend may be reasonable if recent changes are due to (known) transient effects. However, without knowledge of the actual calendar year trends observable in the data, how can a rational decision regarding future calendar year trend emergence be made?

A modelling framework which includes proper estimation of calendar year trends, as well as control over future assumptions, is required.

Most optimal combination of intercepts, trends, and link ratios

The most optimal combination of intercepts, trends, link ratios, and variance weights (based on the Bayes Information Criterion) is shown below. The best link ratio average to use is the arithmetic average (delta = 2). Intercepts are often needed along with constant trends down the accident years (middle column) for four development periods. Link ratios, especially early on, do not have predictive power (the link ratios for 0~1, 2~3, and 3~4 are all optimised to 1).

ABC:PL(C):ELRF[Trend Ratio - D(2)?1]:Regression Table

ELRF Parameter Estimates

Development	Intercept			Trend			Ratio			
Period	Est.	S.E.	P-Value	Est.	S.E.	P-Value	Est.	Ratio-1	S.E.	P-Value
0~1	82,382.92385	3,326.64334	0.00000	15,236.30448	824.48340	0.00000	1.00000	0.00000	0.00000	0.00000
1~2	****	****	****	****	****	****	1.41494	0.41494	0.01080	0.00000
2~3	37,475.55829	2,239.85724	0.00000	8,660.10167	696.41884	0.00002	1.00000	0.00000	0.00000	0.00000
3~4	25,211.24357	1,860.65061	0.00004	6,181.92696	659.99456	0.00023	1.00000	0.00000	0.00000	0.00000
4~5	****	****	****	****	****	****	1.07234	0.07234	0.00475	0.00002
5~6	13,002.79735	1,089.42272	0.00126	3,088.90271	544.30207	0.01084	1.00000	0.00000	0.00000	0.00000
6~7	-8,268.90732	1,869.52034	0.04750	****	****	****	1.05538	0.05538	0.00495	0.00788
7~8	****	****	****	****	****	****	1.02581	0.02581	0.00137	0.00282
8~9	****	****	****	****	****	****	1.02014	0.02014	0.00052	0.01648
9~10	****	****	****	****	****	****	1.01626	0.01626	****	****
To Ultimate							1.00000	0.00000	0.00000	****

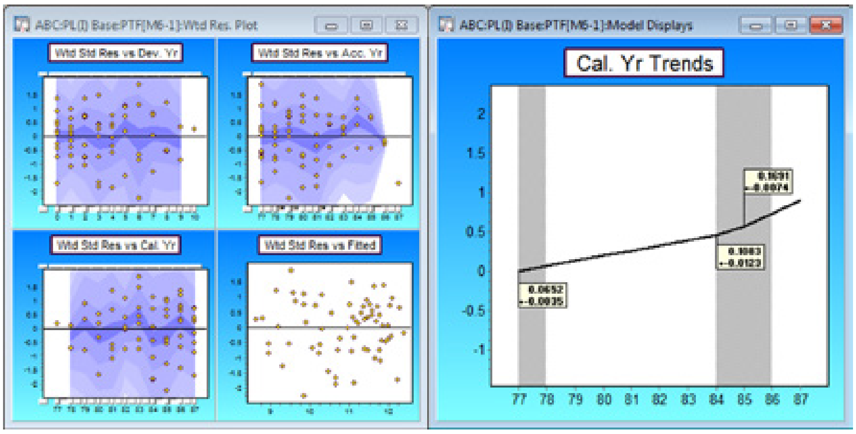
Delta = 2, AIC = 1,044.3
If the test is to be conducted at an overall 5% level, a parameter would be regarded as insignificant if the corresponding P-Value is greater than 0.003414

The model above is quite different to the Mack or Murphy methods, nevertheless the same problem is seen in the calendar direction residuals. The trends it is able to measure are the best option for the generalised ratio framework but they are unable to make up for the inability to measure calendar trends. Projections are still most likely to be too low.

Summary

The link ratio methodology is not able to describe calendar year trend changes. In this example, the early calendar year trends are lower than more recent calendar year trends (in the last three years particularly). The Mack method, along with other link ratio methods, describe an average calendar year trend with the result that the early calendar years are over-fitted and more recent calendar years are under-fitted. The implications of this example is that the next future calendar years will also be underfitted. The best estimate of the mean of the loss distribution obtained from link ratio methods will be too low.

- The Mack and Murphy methods are not able to accommodate or measure calendar year trends,
- The liability stream from the Mack method is significantly lower than could be expected given the recent history by calendar year.
- Knowledge of the historical calendar year trends is required before sound decisions can be made regarding future expectations.
- A different model that is able to parameterise changes in calendar trends is easily able to correctly capture the structure of this data. See below. This kind of model will be described in more detail later in this booklet.



2.3. The Mack method gives a best estimate twice as large as it should be

In this case study, the Mack method is applied to real data: LR High. The residuals versus calendar year demonstrate clear issues with the method. Continuing with the projection we find that the estimated reserve mean is 902M.

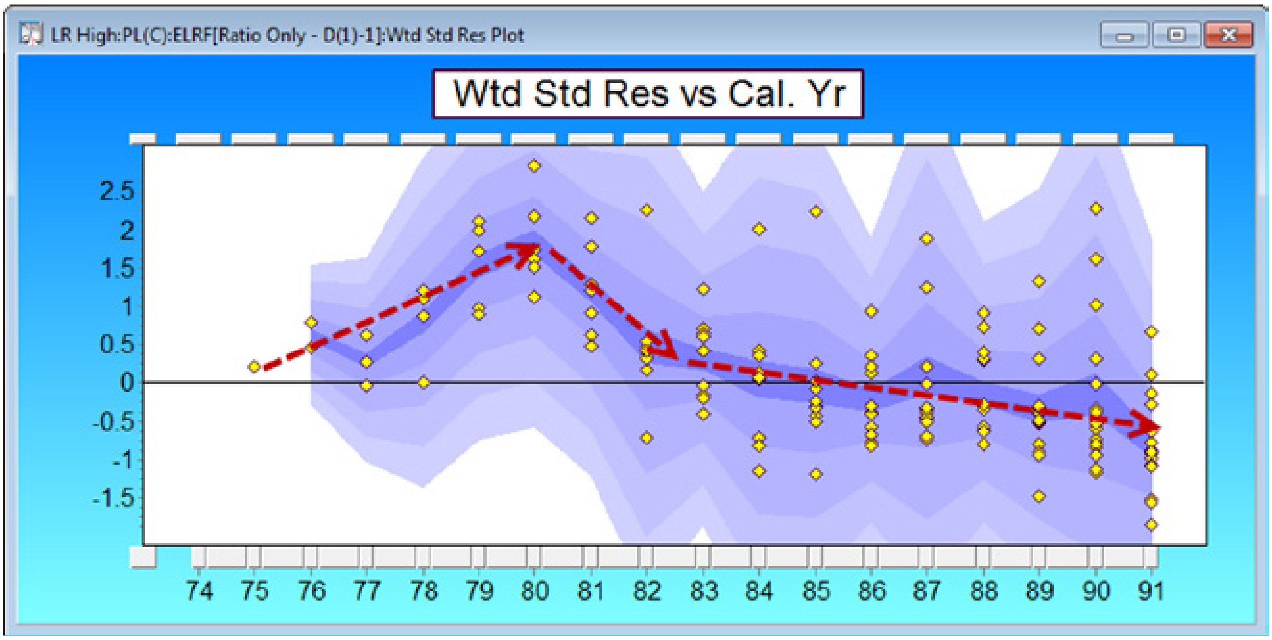
After spending a few minutes addressing the most obvious issues with the method, the optimal combination of intercepts, ratios and trends down the accident years, estimates a new reserve mean of 489M.

The total reserve of 902M is a large amount of money to lock aside for losses given a better estimate of the total losses is 489M.

The link ratio methods do not provide a best estimate of either the mean or the volatility.

Mack method

The residuals versus calendar year for the Mack method are shown below.



The residuals by calendar year show definite changes in trends as indicated by the red arrows. Most recently there is a negative trend in the calendar years. This trend in the residuals means that, everything else being correct, the method Is overestimating the recent calendar year trend.

LR High:PL(C):ELRF[Ratio Only - D(1)-1]:Forecast

Incremental | Cumulative

Accident Period vs Development Period (Incremental Forecast)

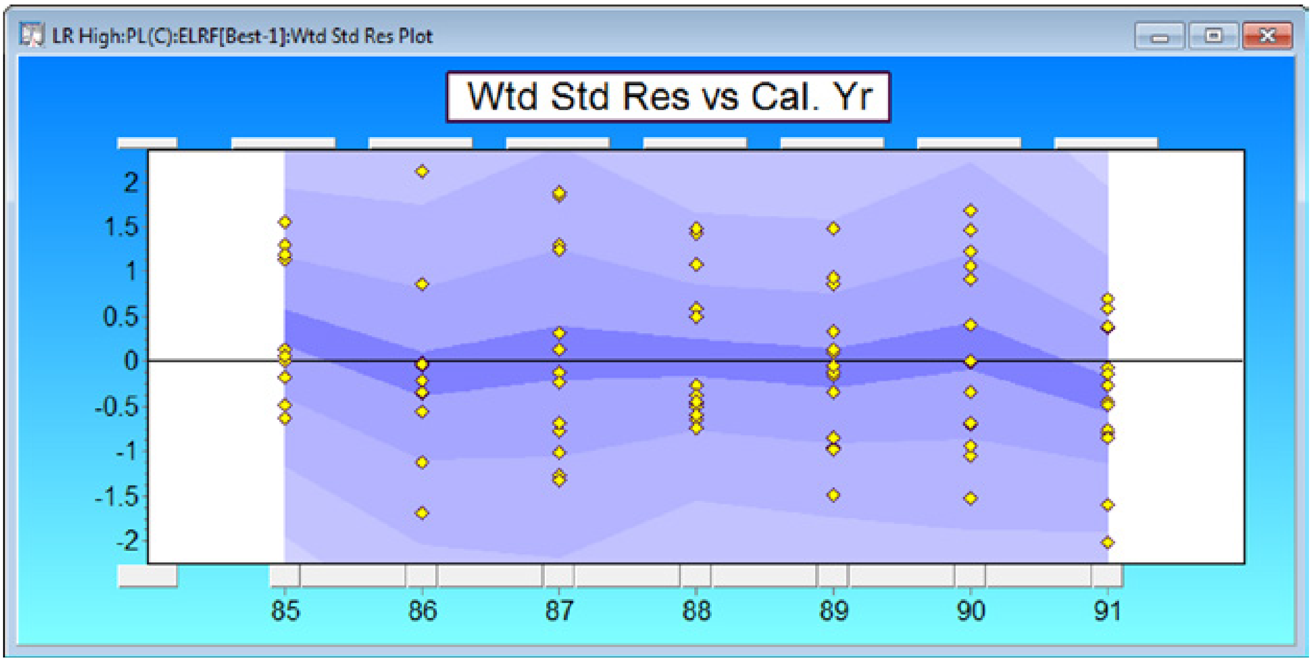
	Cal.Per.Total.	0	1	2	3	4	5	6	7	8	9
1982	86,752	4,385	25,090	29,071	21,520	14,595	8,139	3,361	1,412	556	425
	94,472	4,385	23,755	23,420	18,082	8,758	4,928	1,825	415	225	152
1983	96,606	4,992	28,566	27,449	21,928	15,201	8,769	3,663	1,552	610	466
	97,248	4,992	21,578	25,968	19,998	11,935	6,352	2,792	312	275	312
1984	108,075	5,410	30,955	29,799	22,485	15,281	8,939	3,683	1,531	603	462
	101,938	5,410	23,435	25,028	19,045	13,182	5,220	1,055	372	462	311
1985	112,487	4,805	27,493	28,252	22,285	14,925	8,560	3,563	1,494	596	456
	94,702	4,805	22,542	26,045	17,828	11,235	5,870	1,822	1,144	459	309
1986	112,774	4,905	28,066	33,712	29,080	20,202	11,540	4,637	1,983	790	606
	91,132	4,905	27,728	37,040	26,728	14,752	3,818	3,326	1,339	538	363
1987	114,032	5,822	33,315	46,710	39,754	27,219	15,061	6,459	2,762	1,101	844
	100,965	5,822	39,392	50,032	34,635	15,190	10,324	4,034	1,629	656	445
1988	127,860	8,358	47,820	64,066	54,316	34,685	20,783	8,913	3,811	1,519	1,164
	126,695	8,358	53,658	68,120	35,372	13,000	12,513	4,908	1,988	802	547
1989	163,568	9,618	55,030	88,253	61,805	43,985	26,356	11,303	4,833	1,926	1,476
	160,895	9,618	75,810	62,652	13,871	15,229	14,533	5,721	2,324	940	644
1990	209,517	15,225	87,115	86,241	70,838	50,413	30,208	12,955	5,539	2,207	1,692
	216,415	15,225	68,255	14,193	16,158	17,063	16,005	6,325	2,577	1,043	719
1991	288,668	13,628	77,974	94,632	77,730	55,318	33,147	14,215	6,078	2,422	1,857
	201,775	13,628	11,437	19,084	19,645	19,337	17,462	6,941	2,840	1,150	800
	Total Fitted/Paid		1992	1993	1994	1995	1996	1997	1998	1999	2000
Cal. Yr Totals	1,625,017		283,445	240,246	167,934	102,867	53,904	23,595	10,734	5,356	3,342
	1,625,018		****	****	****	****	****	****	****	****	****

1 Unit = $1,000

Of the last eight calendar years, the fitted totals are higher for seven – for the last calendar year in particular. Examination of the future liability stream (1992~1994) provides immediate visual indications that the projections are far too high.

Optimal intercept, trend, and ratio model

For an optimal combination of intercepts, trends, and ratios, the residuals versus calendar year are shown below. Note only the data from the last seven calendar years was used in the model fitting.



The model has clearly better described the calendar year trends – however we still have no idea what the trends are.

LR High:PL(C):ELRF[Best-1]:Forecast

Incremental | Cumulative

Accident Period vs Development Period (Incremental Forecast)

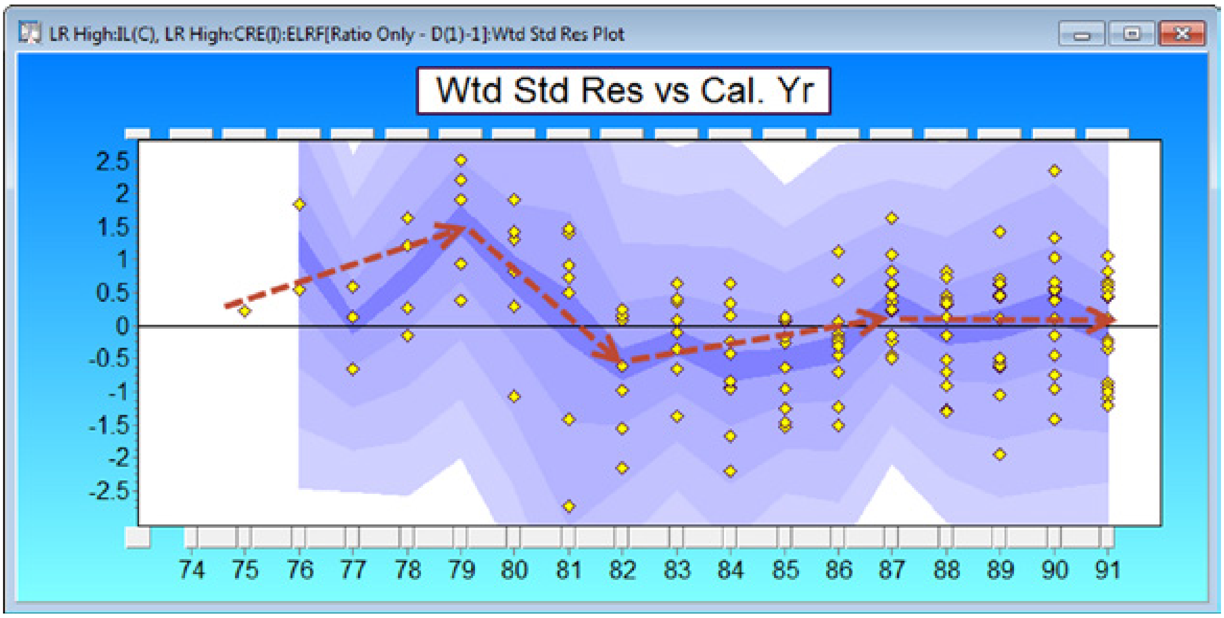
	Cal.Per.Total.	0	1	2	3	4	5	6	7	8	9	Outstanding	Ultimate
1982	46,498	4,385	4,534	17,045	17,956	11,418	5,112	2,165	880	393	364	113	86,058
	94,472	4,385	23,755	23,420	18,082	8,757	4,928	1,825	415	225	152	280	280
1983	60,282	4,992	10,867	21,740	18,297	11,418	5,112	2,165	880	393	364	408	94,610
	97,248	4,992	21,578	25,968	19,997	11,935	6,352	2,793	313	275	318	445	445
1984	71,632	5,410	17,199	26,434	18,762	11,418	5,112	2,165	880	393	364	724	93,472
	101,938	5,410	23,435	25,028	19,045	13,182	5,220	1,055	372	275	314	525	525
1985	84,315	4,805	23,532	31,129	18,594	11,418	5,112	2,165	880	393	364	1,527	91,675
	94,702	4,805	22,542	26,045	17,828	11,235	5,870	1,823	614	270	309	808	808
1986	94,768	4,905	32,580	39,080	24,264	12,456	5,576	2,362	960	428	397	3,958	118,928
	91,132	4,905	27,728	37,040	26,728	14,753	3,817	956	787	346	396	1,408	1,408
1987	109,405	5,822	42,779	47,885	33,171	13,494	6,041	2,558	1,040	464	430	10,259	155,331
	100,965	5,822	39,392	50,032	34,635	15,190	1,487	1,240	1,019	447	513	2,352	2,352
1988	129,668	8,358	54,129	57,544	45,321	14,532	6,506	2,755	1,120	500	463	25,500	191,007
	126,695	8,358	53,658	68,120	35,373	3,815	1,832	1,520	1,247	548	628	4,787	4,787
1989	156,597	9,618	62,189	63,519	51,570	14,532	6,506	2,755	1,120	500	463	76,998	225,078
	160,895	9,618	75,810	62,652	6,000	4,572	2,166	1,788	1,464	642	737	8,278	8,278
1990	189,785	15,225	70,249	69,494	53,275	14,532	6,506	2,755	1,120	500	463	148,105	231,585
	216,415	15,225	68,255	13,538	7,800	4,731	2,236	1,844	1,509	662	759	20,174	20,174
1991	215,769	13,628	72,715	70,079	54,475	13,494	6,041	2,558	1,040	464	430	220,719	234,347
	201,775	13,628	15,270	14,354	9,692	4,843	2,274	1,871	1,530	671	770	29,604	29,604
	Total Fitted/Paid		1992	1993	1994	1995	1996	1997	1998	1999	2000	Total Reserve	Total Ultimate
Cal. Yr Totals	1,024,151		218,464	148,706	80,063	24,625	10,623	4,295	1,526	373	-211	489,017	2,114,034
	1,625,018		****	****	****	****	****	****	****	****	****	40,316	40,316

1 Unit = $1,000

With the most optimal model in this framework, the estimate of the total reserve is now 489M!

What about the Incurred losses?

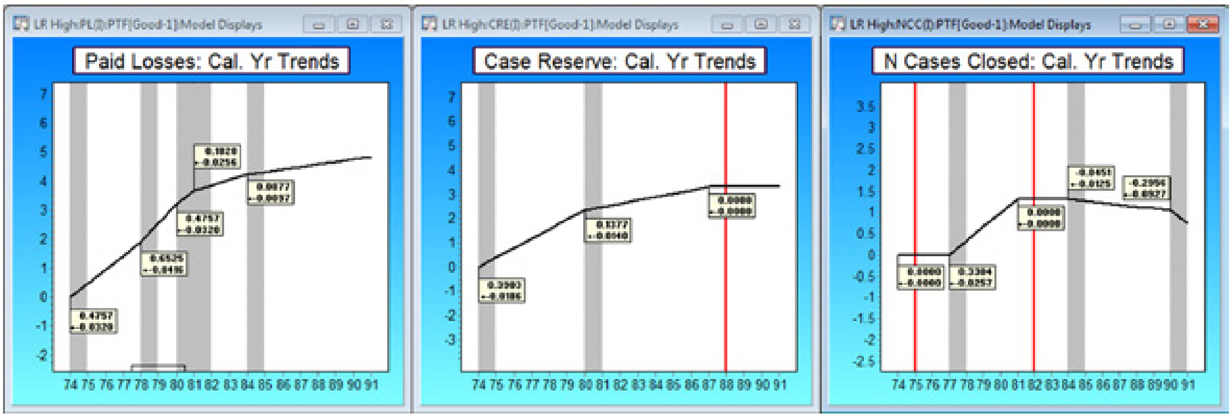
Applying the Mack method to the Incurred losses also demonstrates problems with the calendar years.



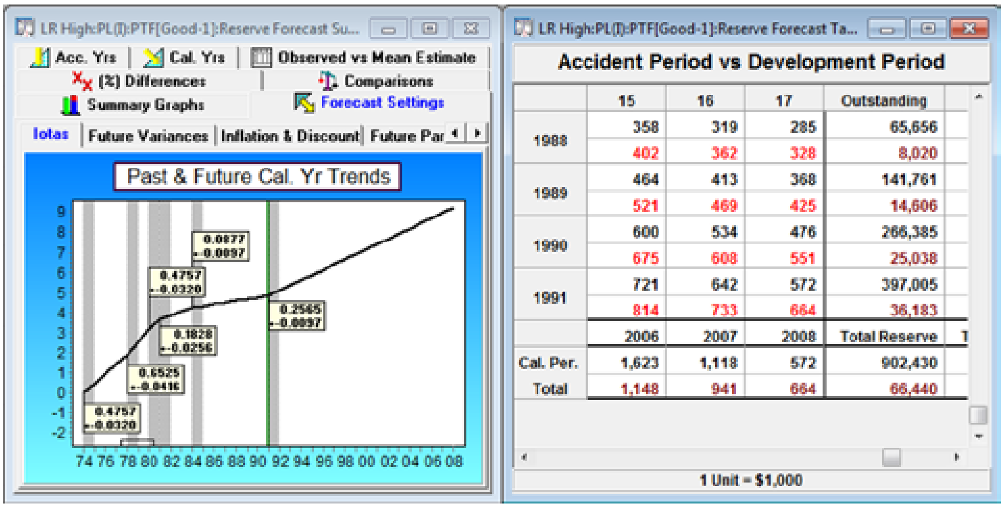
Although the most recent calendar year residuals appear better, the method clearly does not describe the changes in calendar trends in the prior years. Further, incurred losses cannot provide a liability stream.

The forecast for this method is 469M. Without recourse to measuring all the trends in the data (development, accident, and calendar), how would you choose the best estimate of the total reserve?

Below are the calendar year trends measured in the PTF modelling framework for the Paid Losses (left), Case Reserve Estimates (center), and Number of Cases Closed (right). The information gleaned from these models can then be utilised in making informed decisions regarding future calendar year assumptions. From the displays, we can see the future calendar year trends in the paid losses are unlikely to increase in the near future since the trends in the Case Reserve Estimates have been zero since 1988 and the Number of Cases Closed are decreasing since 1984 with a further major drop between 1990 and 1991. In fact, from this information we could hypothesise that the trends in the paid losses are likely to decrease further.



In order to reach the mean reserve projected by the Mack method, 902M, what future calendar year assumptions are required in the identified PTF model for the paid losses?



A future calendar year trend of 25.65%+_ must be assumed for all future calendar years. This trend is nearly three times higher than the most recent measured trend which, if we examine the case reserves and number of cases close, **we expect to decrease.**

Summary

Ratio methods are found to be unsuitable for the LR High data.

- Calendar year trend changes are evident in the data,
- The Mack method overestimates the calendar year trend with the result that the future projections are far too high,
- Models for incurred losses give much lower forecasts. How do you decide which is the more correct estimate of the total reserve mean?
- In contrast, the identified PTF model measures trends and volatility about the trends. Future trend assumptions are under the actuary's control.

3. When can accident years be treated as development years and vice versa?

The two time dimensions, accident and development, will naturally have very different emerging experiences. Since the experience is different, conditioning on accident years should produce different mean estimates than conditioning on development years.

In this case study we demonstrate:

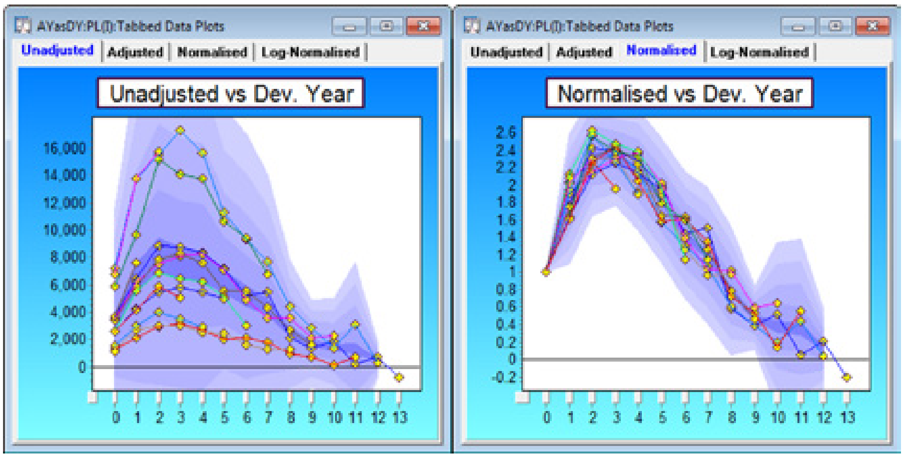
- Chain Ladder (Mack) link rato methods forecast the same mean incrementals irrespective of the direction conditioned due to the nature of the calculations. That is, the total mean reserve is the same.
- the Chain Ladder (Mack) volatility estimates are dependent on the direction conditioned. These estimates can vary wildly. Which volatility estimate is correct?

These characteristics illustrate that link ratios are oblivious to the underlying trends in the data. They are thus incapable of solving the reserving (and pricing) problem.

3.1 Chain-ladder method is specified for a different problem.

Development ratios embody the intuition that for each line of business the pattern in which the claims develop is more or less the same accident year by accident year. According to this idea, each accident year has its own level of business but once you correct for that the development pattern is seen to be unchanging beneath the intrinsic volatility of the kind of data involved.

This is illustrated by the graphs below, which use simulated data. On the left side incremental payments are plotted against development year with coloured trace lines partitioning the results by accident year. On the right the same data has been normalised to create parity in the first payment for each accident year. The underlying pattern stands out clearly and can be used to estimate the future losses.



We have used incrementals to make the pattern stand out more clearly but the same concept underlies the use of ratios for cumulatives.

The nature of the reserving problem calls for different treatment of accident and development axes.

The credibility of this methodology rests on an understanding of the forces that shape this kind of data. It implies that the development and accident directions represent distinct axes of change in which there is a strong dependence between consecutive results in the development direction and little or no such relationship in the accident direction.

Models for this kind of data should therefore not be symmetrical relative to the interchange of development and accident axes.

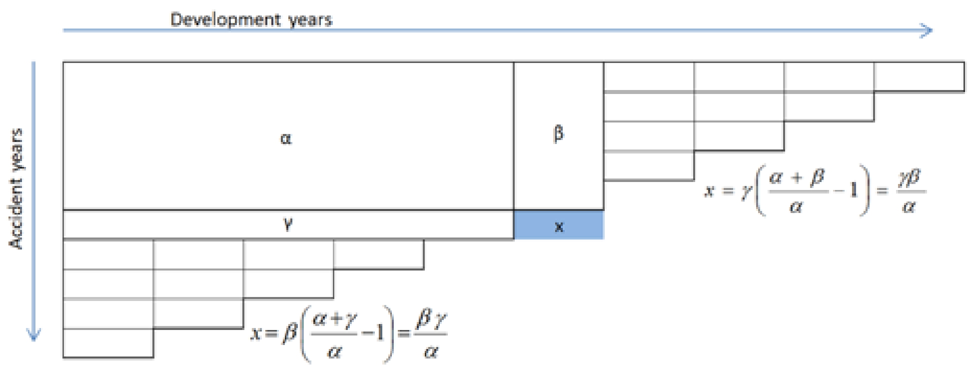
In one case this symmetry does exist. Unfortunately, it happens to the most popular of all ratio methods the Mack method (also know as: chain-ladder or volume-weighted averages method). It is not intended in the formulation of the model, but arises as a computational artefact. The cell-by-cell (incremental) forecasts of the model applied to transposed data are identical to those for the original data.

In this case the model therefore contains a symmetry where there is none in the state of affairs that is being modelled. This should rule the model out of consideration from the very start.

3.2 The source of the symmetry

The symmetry in the chain-ladder method is not apparent in its usual description but is very easy to see when the method is looked at in the right way.

In the diagram below each cell represents the corresponding incremental. The Greek letters are the sums of the numbers in the corresponding cells. The cumulative corresponding to any cell is the sum the incremental for that cell and all incrementals to the left of it. Thus γ is the cumulative immediately preceding x, and α is the sum of all the cells in the rectangle.



The chain-ladder calculation for the future incremental cell x proceeds as follows: multiply the previous cumulative $(=\gamma)$ by the link ratio minus one. The chain-ladder link ratio is $(\alpha+\beta)$ divided by α, and so the link ratio minus one is just β/α. Therefore $x = \gamma(\beta/\alpha) = (\gamma\beta)/\alpha$.

It is now obvious that if the same calculation is performed with the development and accident axes interchanged, that is forecast development period we compute the cumulative across the accident years, the result will be identical. Here the ratio is $\alpha + \gamma$ divided by α.

Of all models used for insurance loss triangles the chain-ladder is the *only one* that has this symmetry. It is analogous to limiting the transition matrix for a Markov Chain model to being doubly stochastic, when nothing in the problem calls for it. It is a hidden and unintended artefact of the choice of model which effectively constrains the forecasts in a way that is irrelevant to the nature of the problem.

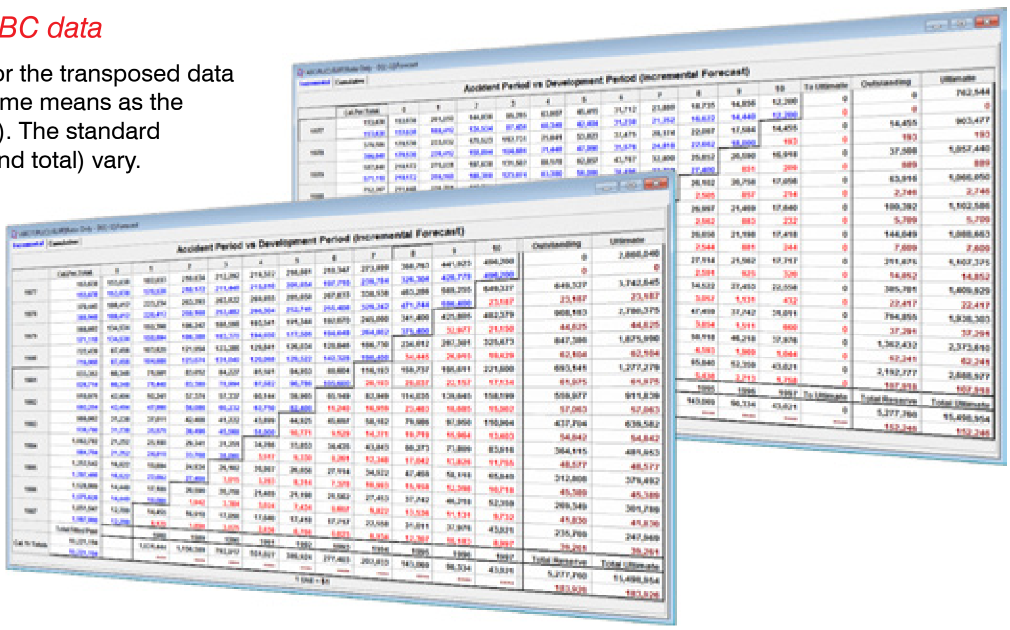
3.3. What happens to the volatility?

The Mack method is the regression version of the chain-ladder. It produces a forecast mean identical to that of the chain ladder and hence is also limited by the same symmetry in this respect. Being 'stochastic' it also associates a standard deviation with each of its forecasts. Are these also the same? The answer is no. The Mack regressions are conditioned on the initial vector, development period zero when done in the usual way, and first accident period when transposed version. Since these are different the details of the regressions are different. Thus Mack produces the same mean forecasts in two different ways but two entirely different measures of variability.

Which measure of the volatility is correct? The method is applied to the same data – just presented differently. Both results provide estimates of the volatility in the data. Which estimate is more accurate? Is there any way to tell?

Projections for ABC data

The Mack method for the transposed data (below) gives the same means as the original data (above). The standard deviations by cell (and total) vary.



4. Bootstrapping link ratio methods – the silver bullet?

The bootstrap technique provides a mechanism for obtaining a distribution of a sample statistic (for example: the mean), where parametric distribution assumptions for the error term, ε, are inappropriate or cannot be tested due to a small sample size. In the loss reserving context the technique can be extended to estimate distributions of forecast random variables. It can also be used to test a model. The model is misspecified if the bootstrap samples do not have the same salient statistical features as the original data.

Many practitioners refer to a bootstrap model. This is a misnomer. The bootstrap is a technique or algorithm used to estimate certain statistics belonging to an already-fitted model. In the actuarial setting the underlying model is generally Mack's model, and the bootstrap is extended with a number of ad hoc additions so that forecast distributions can be derived.

As a case in point, the readings for the 2013 CAS Exam 7 includes an article describing such a bootstrap model, Shapland and Leong (2010). The model uses Over-Dispersed Poisson (ODP) residuals to bootstrap the Mack method, and draws additional 'residuals' from a Gamma distribution to cover the process variability in the cells in the forecast period. In this case doing whatever it takes to produce an acceptable-looking outcome takes precedence over gaining an understanding of the volatility of loss reserves, to the ultimate detriment of the actuarial profession. The basic bootstrap algorithm is as follows:

- Fit a model to the data (for instance, the Mack method),
- Calculate the statistic of interest,
- Calculate the residuals (difference between the data and the method),
- Resample the residuals and create a new sample of data, known as pseudo-data,
    - NB: the residuals are assumed to be independent and identically distributed.
- Repeat step 1 N times until a sufficiently large sample is obtained for calculation of the statistics of interest.

A number of key observations about the technique:

- The residuals used to generate the bootstrap sample are understood to come from fitting the model in question.

It does not make any sense to fit one model, then use error terms from another as considered in Shapland and Leong (2010). If the two models are not identical, then total volatility (variation explained + variation not explained) cannot equal the variation in the data. Further, it is impossible to determine whether combining the models in this way introduces more volatility or reduces the volatility. This knowledge is critical to determining the value of any inference!

- When applied correctly, there is no need to introduce other assumptions about the distribution of the residuals.

If the residuals contain structure, then the assumption that all residuals are identically distributed is not satisfied. The pseudo-data would then significantly differ from the original data in respect of the statistics of interest.

When the data has a complex structure such as in the loss reserving problem, the bootstrap technique is best used as a diagnostic tool to aid inference. While the technique cannot compensate for a poor model, it will show if a model is deficient. If the underlying model does answer to the processes that generated the data, the bootstrap will fail to provide sound inference regarding any statistic of interest.

The bootstrap does not provide a silver bullet to deal with all the deficiencies of the ratio based methods. Rather, it amplifies the discrepancies in the link ratio methods (including Mack) and introduces false confidence by appearing to introduce stochastic elements.

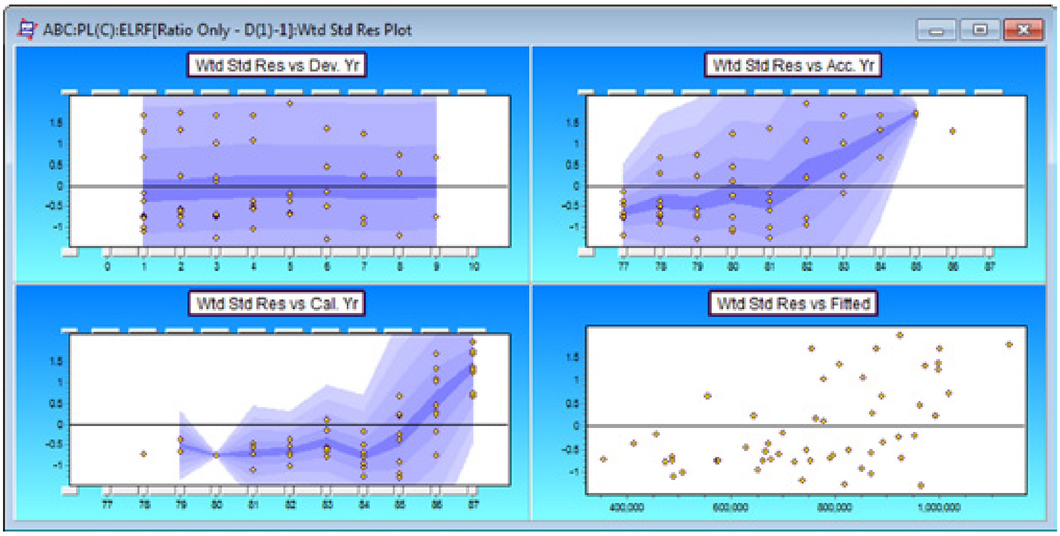
When structured modelling is ruled out due to lack of data and/or complete ignorance of the generating process the bootstrap is the best recourse for inference. The loss reserving problem, with loss reserves estimated via link ratio methods, is not in this class.

4.1. Company ABC

The bootstrap technique is applied to the Mack method fitted to the ABC data. As noted previously, the losses in this line of business demonstrate strong calendar year trends not described by the Mack method. The bootstrap is not able to correct for the poor model. Rather, the bootstrap provides a clear diagnostic that the model is deficient.

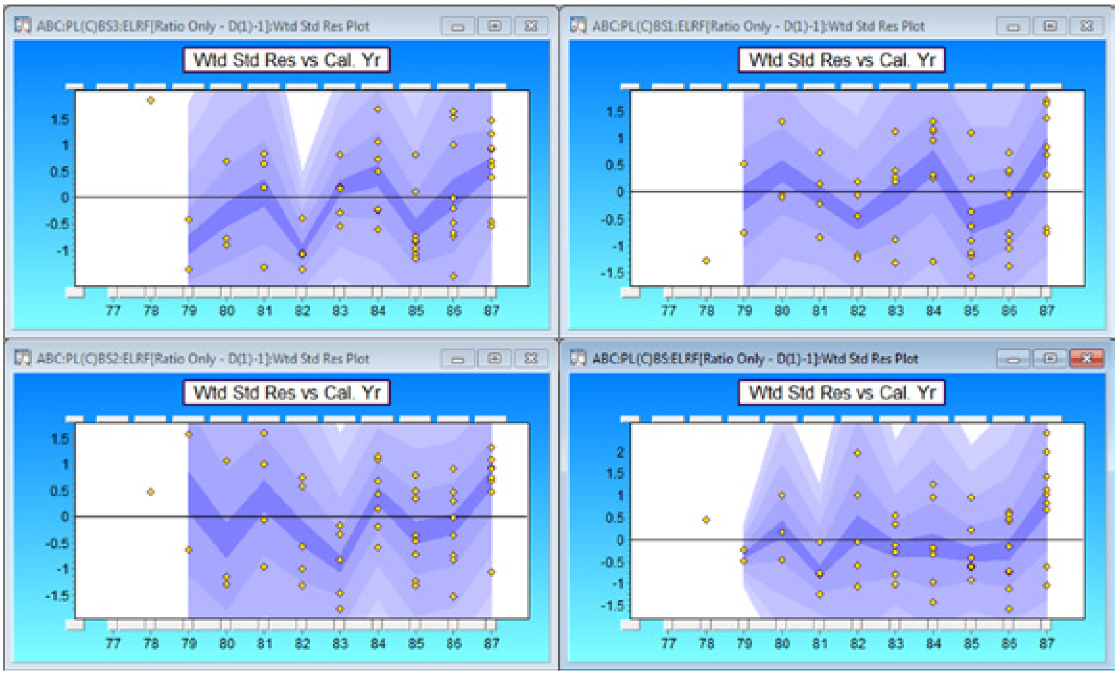
The residuals used as input into the bootstrap algorithm are shown below. Note that these residuals are from the Mack method itself.

Mack residuals



There are obvious patterns in the Mack residuals above. In particular, the calendar year residuals are almost all negative between 1977 and 1984. In 1987, all the calendar year residuals are positive.

Residuals from the Mack method applied to four bootstrap samples are illustrated below against calendar years.



A necessary but not sufficient, condition for the bootstrap technique to work is that the residuals be randomly distributed around zero, for all three time directions.

The trends clearly identified in the real data have disappeared from the bootstrap samples. Since the method does not describe the calendar year features, the bootstrap samples destroy the evidence. As a result, the projections from the bootstrap samples differ wildly from the Mack method mean. However, neither the Mack method mean nor the means from the Mack method applied to the bootstrap samples have anything to do with the paid losses. The method does not describe the data.

We continue with the bootstrap simulations to a total of 10000 bootstrap samples to show the sample means arising from the bootstrap technique. Note that the residuals from the Mack method are used.

The following diagnostics are obtained by accident year and calendar year.

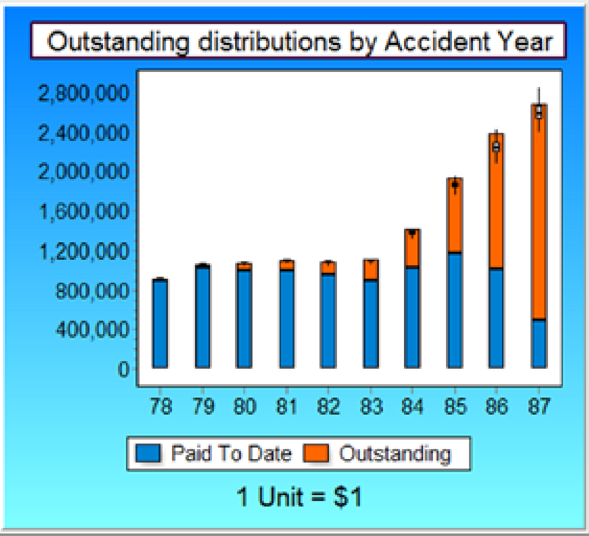
ABC:PL(C):ELRF[Ratio Only - D(1)-1]:Bootstrap Forecast

All Statistics | Acc Year: Total | Simulated Values | Quantile Summary | Settings |

Sample-Based Statistics

Accident Year	Sample Mean	ELRF Mean	Sample Median	Sample S.D.
1978	13,950	14,455	13,881	910
1979	35,867	37,508	36,005	1,966
1980	59,927	63,916	59,923	3,598
1981	94,108	100,392	93,506	5,779
1982	130,620	144,049	130,400	8,982
1983	193,384	211,675	191,540	14,859
1984	352,005	385,701	350,397	23,235
1985	692,636	764,855	690,673	41,567
1986	1,231,581	1,362,432	1,226,919	78,281
1987	2,099,761	2,192,777	2,087,690	103,399
Total	4,903,838	5,277,760	4,895,891	156,788

10,000 Simulations. 1 Unit = $1



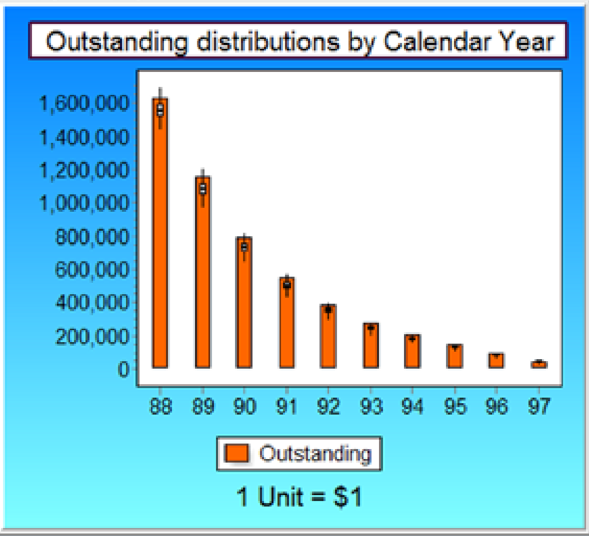
ABC:PL(C):ELRF[Ratio Only - D(1)-1]:Bootstrap Forecast

All Statistics | Cal Year: Total | Simulated Values | Quantile Summary | Settings |

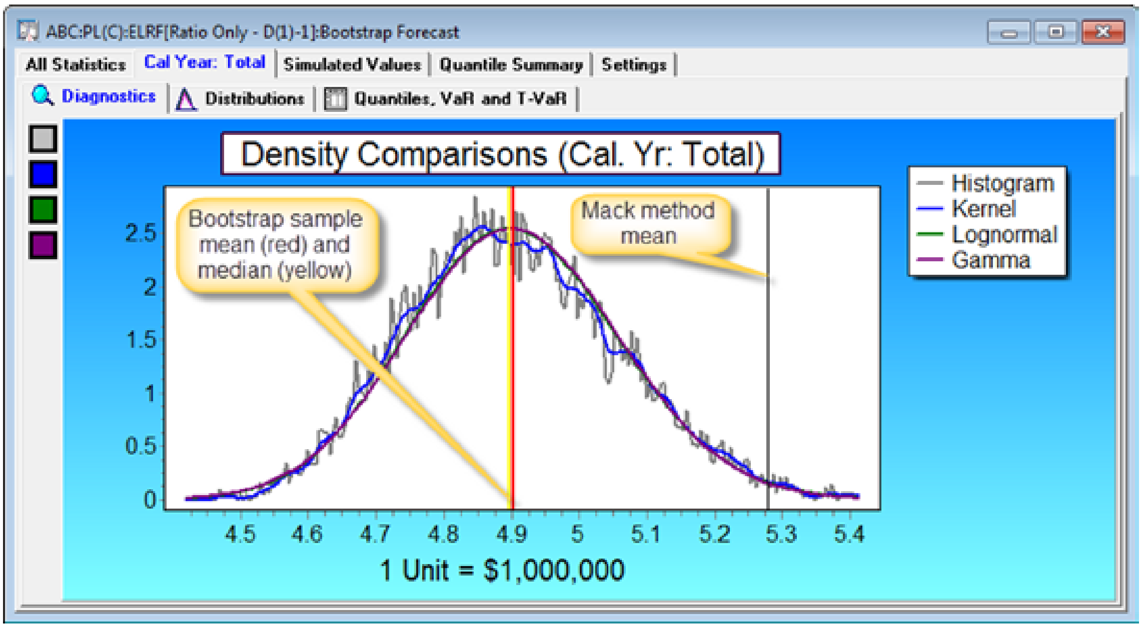
Sample-Based Statistics

Calendar Year	Sample Mean	ELRF Mean	Sample Median	Sample S.D.
1988	1,558,763	1,631,444	1,554,798	56,155
1989	1,084,211	1,156,589	1,082,161	51,491
1990	730,178	792,917	728,987	37,820
1991	501,979	551,027	500,438	29,327
1992	350,696	389,924	348,935	23,093
1993	245,866	277,403	245,826	16,654
1994	181,745	202,033	181,218	10,504
1995	128,036	143,069	128,100	8,191
1996	82,246	90,334	82,389	5,144
1997	40,118	43,021	40,017	2,846
Total	4,903,838	5,277,760	4,895,891	156,788

10,000 Simulations. 1 Unit = $1

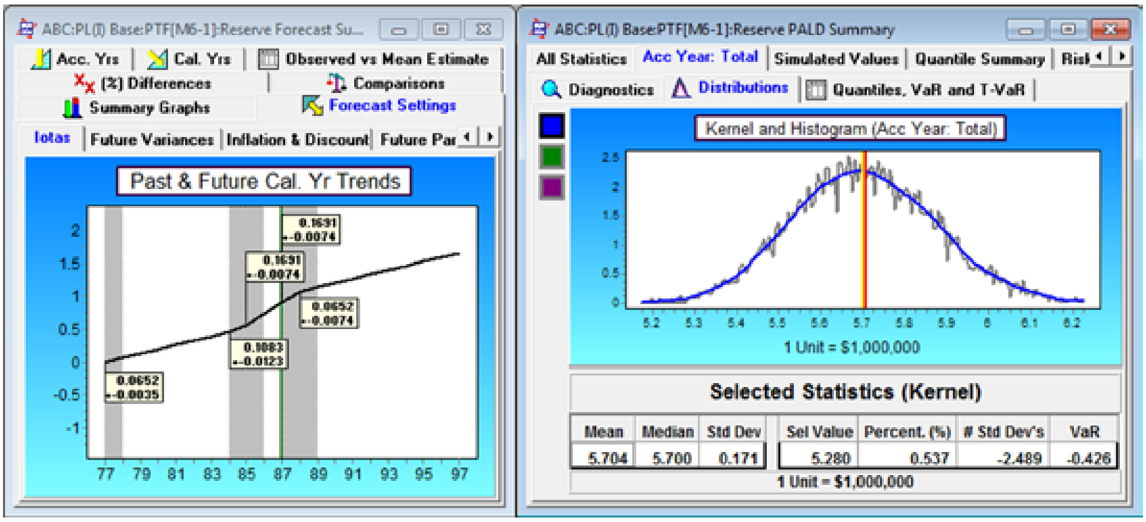


The calendar year display shows the bootstrap samples almost always project lower than the Mack projections. This is due to the high residuals in the last calendar year being placed in early calendar years and low residuals from the early calendar years being placed in the most recent calendar years. Since the redistribution is random, the trend to high values seen in the last calendar years in the original data is lost in the pseudo-data.



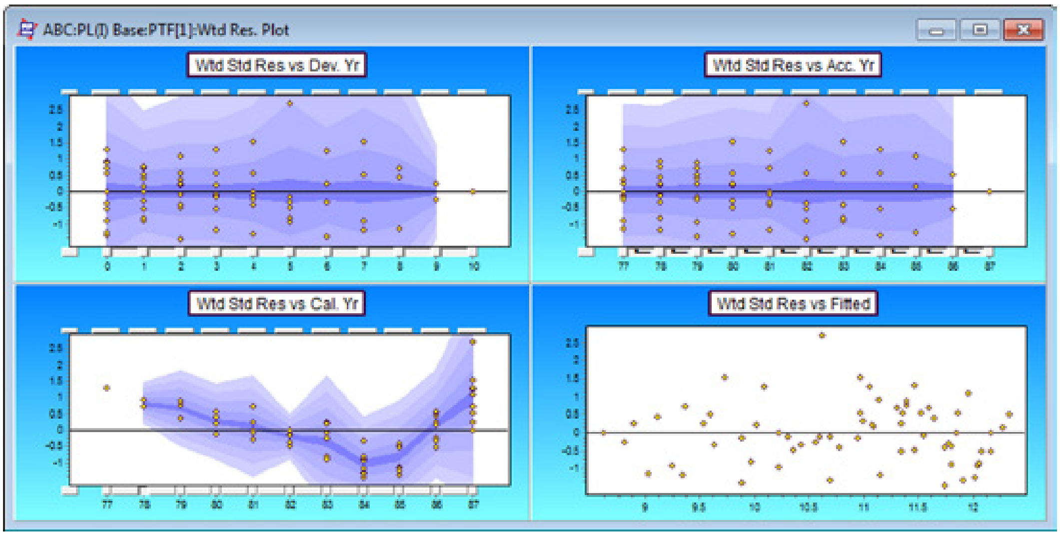
Rather than addressing the problem caused by the calendar year trends, application of the bootstrap has made the situation worse. The Mack method mean was already far too low (calendar year residuals shown previously). The mean of the bootstrap samples is substantially lower than the mean obtained from the Mack method.

The final calendar year trend, measured in the Probabilistic Trend Family (PTF) modelling framework is 16.91%+_0.7%. An optimistic forecast scenario incorporating this measurement results in a projected total mean reserve of 5.71M. The Mack mean, at 5.28M is 7% lower than this optimistic scenario.

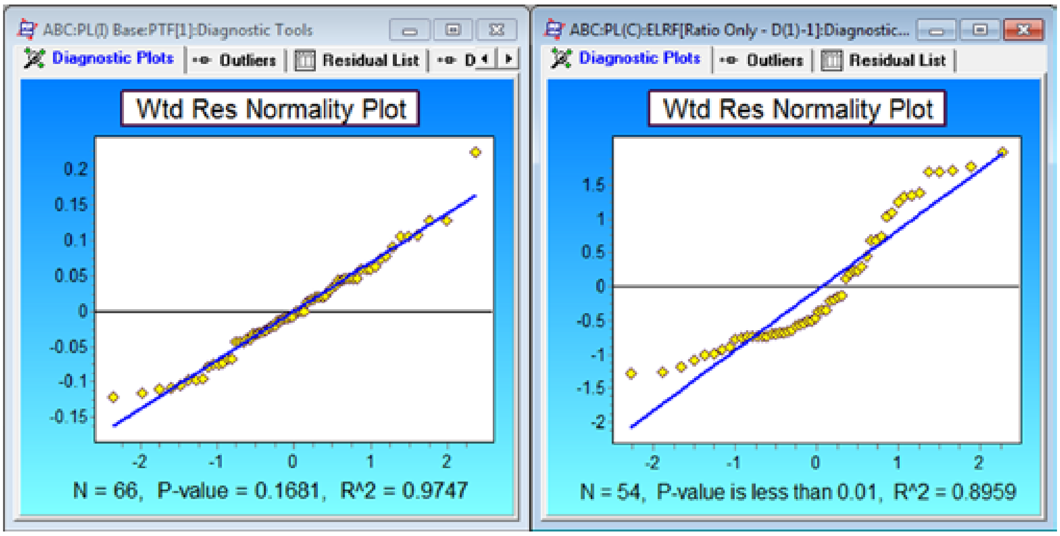


The distribution on the right shows the Mack method mean of 5.28M on the distribution graph. In order for this mean to be reached, if the losses actually followed the distributions projected from the PTF model, then the probability of observing the Mack mean (or under) is less than 0.54%!!

Over-Dispersed Poisson residuals



Not only do the ODP residuals exhibit different structure to the Mack method residuals, they clearly come from a different distribution overall. The ODP residuals (left) are shown in contrast with the residuals from the Mack method (right).



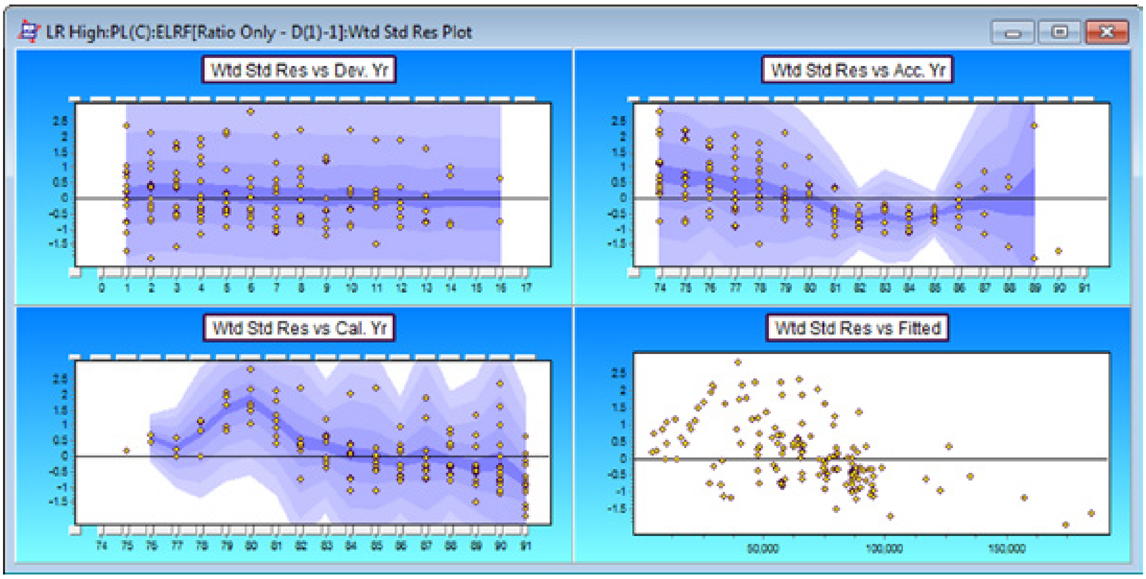
In addition, the ODP model does not condition on development period zero thus more residuals are present in the ODP model (66 versus 54).

Some actuarial software products bootstrap the Mack method with residuals from the Over-Dispersed Poisson model (along with other ‘enhancements’). Simulations created in this mixed way do not represent anything in particular, much less do they provide a basis for reliable forecasts.

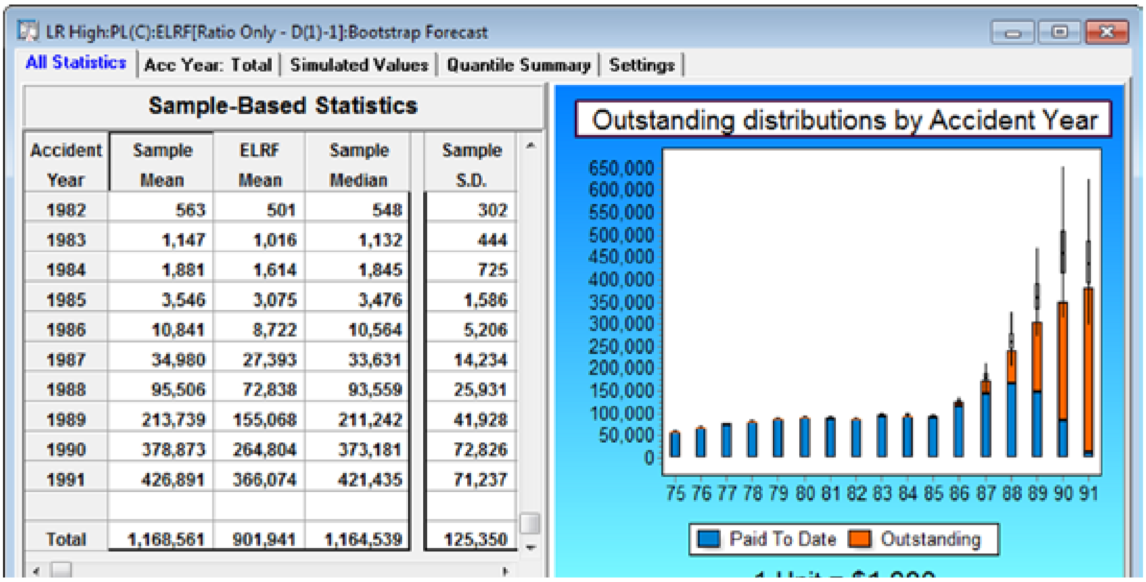
4.2. Company LR High

The bootstrap technique is applied to the Mack method fitted to the LR High data. This illustrates another way in which data can be beyond the modelling capabilities of the method. In this case, the Mack method overestimates the calendar year trend with the result that the projections from the Mack method are far too high.

Mack residuals



Note that the high values are likely to have low residuals. This means that when the bootstrap is applied, these high values are likely to have high residuals (thus inflating the answers). We expect the bootstrap to produce results even higher than the Mack method for this data and model. The display belows shows this - especially for the most recent accident years.



Sample-Based Statistics

Accident Year	Sample Mean	ELRF Mean	Sample Median	Sample S.D.
1982	563	501	548	302
1983	1,147	1,016	1,132	444
1984	1,881	1,614	1,845	725
1985	3,546	3,075	3,476	1,586
1986	10,841	8,722	10,564	5,206
1987	34,980	27,393	33,631	14,234
1988	95,506	72,838	93,559	25,931
1989	213,739	155,068	211,242	41,928
1990	378,873	264,804	373,181	72,826
1991	426,891	366,074	421,435	71,237
Total	1,168,561	901,941	1,164,539	125,350

4.3. Summary

These two case studies serve to illustrate two opposite effects of applying the Mack method without considering whether the method is appropriate to the data.

In the first example, the method produced answers that were far too low with the result that, should the method be used for setting reserves or pricing future accident years, the projections would be substantially under the required reserves. The line would not be profitable.

In the second study, the reserves were vastly overstated resulting in capital being tied up which could have been used more efficiently in other ways. Further pricing based on this method would be overstating the loss costs resulting in the company being less competitive.

Neither situation is optimum for these lines of business. Without the right tool to model the data, how do you know if your company is in the first situation or the second?

5. Link ratio methods introduce spurious process (volatility) correlation measures and do not distinguish between common drivers and process correlation

Typically correlations are measured from the industry and these figures used as benchmarks or specifications for companies to use when calculating the correlations between their own LOBs. As discussed in the brochure, Understanding correlations and common drivers, these calculations are not relevant for individual companies.

Relevance aside, how should these correlations be measured and does the method of calculating the correlations identify the true process correlation?

As illustrated in the brochure, Understanding correlations and common drivers, spurious process correlation is measured when methods fail to de-trend long tail liability data in the three time directions – development year, accident year, and calendar year. In the examples considered, once these trends are accounted for, the volatility (process) correlation is statistically insignificant.

5.1. Paid losses: Industry CAL and Industry PPA

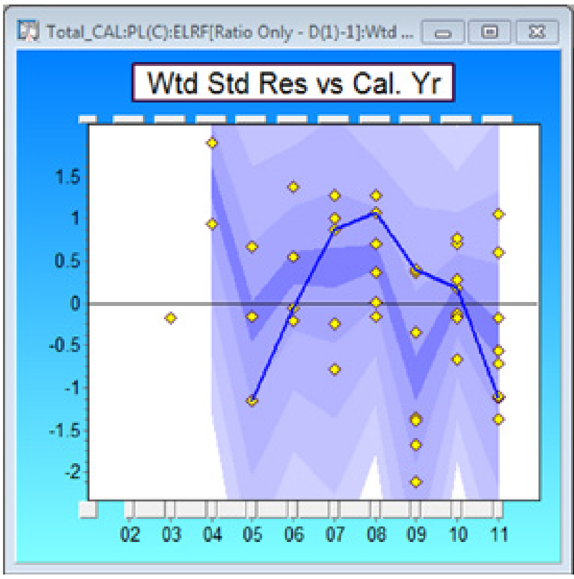
A.M. Best Schedule P data (2011) are used to compare CAL and PPA for two companies, LMI and TG, with each other and the industry. The Industry may expect CAL and PPA to be highly correlated since both lines relate to automobile liabilities and may have common drivers.

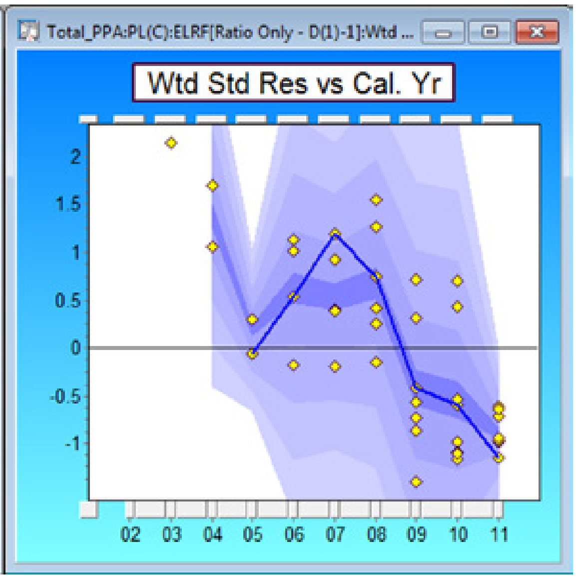
Volatility correlation is model dependent since this correlation is only interpretable relative to mean projections. If the model does not fully de-trend the data then statistically significant volatility correlation may be identified purely as a result of trends remaining in the data. This volatility correlation measure is spurious.

As illustrated in previous case studies, link ratio methods are not able to de-trend the data along the calendar years in the event of changing calendar year trends. Volatility correlation measured between two (or more) lines of business modelled using the Mack method is expected to be statistically significant and very high if common calendar year drivers (trends) are present.

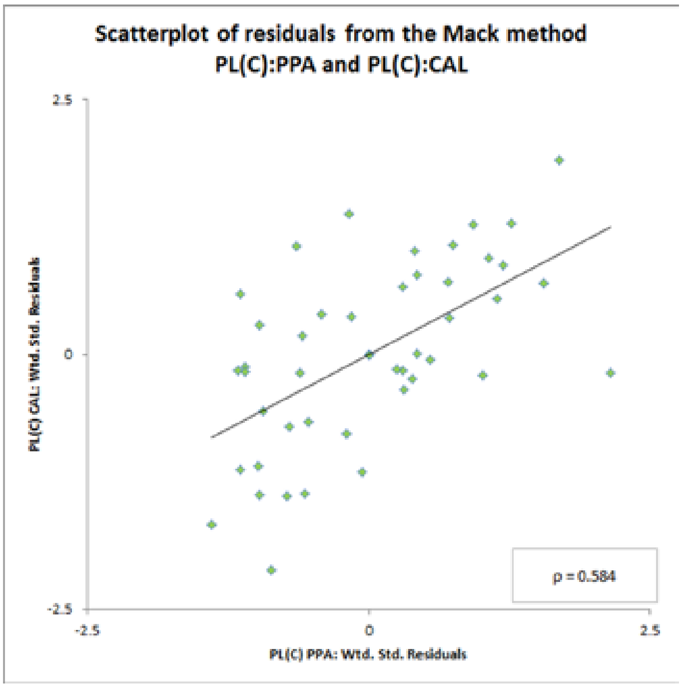
In the following case study, the Mack method is applied to Industry CAL data (CAL) and Industry PPA data (PPA) extracted from A.M. Best (2011) Schedule P data.

The residuals are shown by calendar year for CAL (left) and PPA (right). The marked observations (blue trace line) correspond to the trace for all observations occurring in accident year 2004.



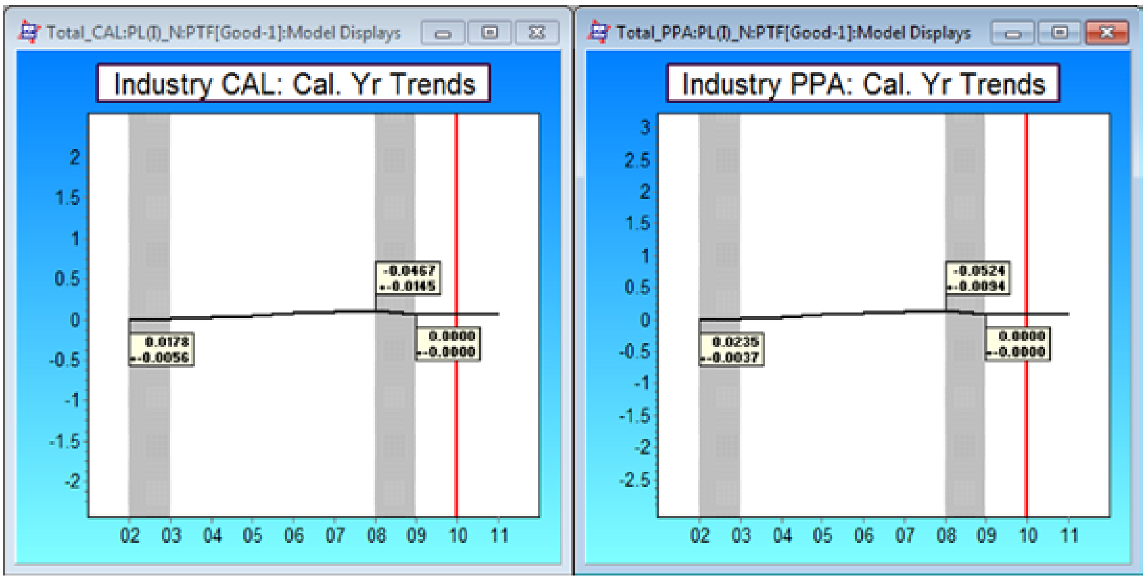


The marked residuals for the Mack method exhibit correlation (by eye); the direction of the trace line changes are similar. This correlation is then measured and shown in the scatter plot of the respective residuals for CAL and PPA below.



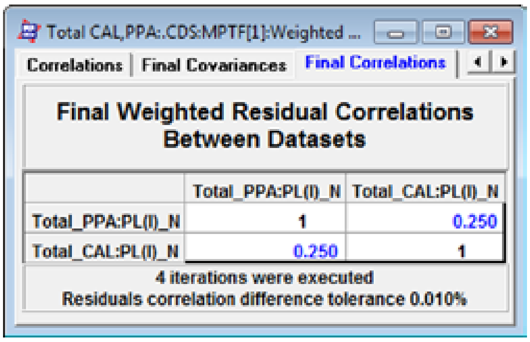
The high volatility correlation of 0.584 may reinforce the common perception that CAL and PPA are related, but is the measure genuine? Both residual displays above demonstrate by calendar year that the Mack method is over-estimating the average calendar year trend. This is identified from the clear negative trend in the residuals over calendar time in both portfolios. Could this common over-estimation of the average calendar year trend be driving the high process correlation measure?

A model is designed in the Probabilistic Trend Family (PTF) modelling framework for both Industry portfolios. The trends identified in the calendar year direction are shown below:



Note the locations of the calendar year trend changes are in exactly the same location – this does indicate common drivers – but the magnitude is different. This difference in magnitude of the trends is an explanation for why the average trends measured in the Mack methods are different, but structurally similar.

Once the common drivers are identified above so the data are fully de-trended in all three directions, the volatility correlation can be measured.



Total CAL,PPA:.CDS:MPTF[1]:Weighted ...

Correlations | Final Covariances | Final Correlations

Final Weighted Residual Correlations Between Datasets

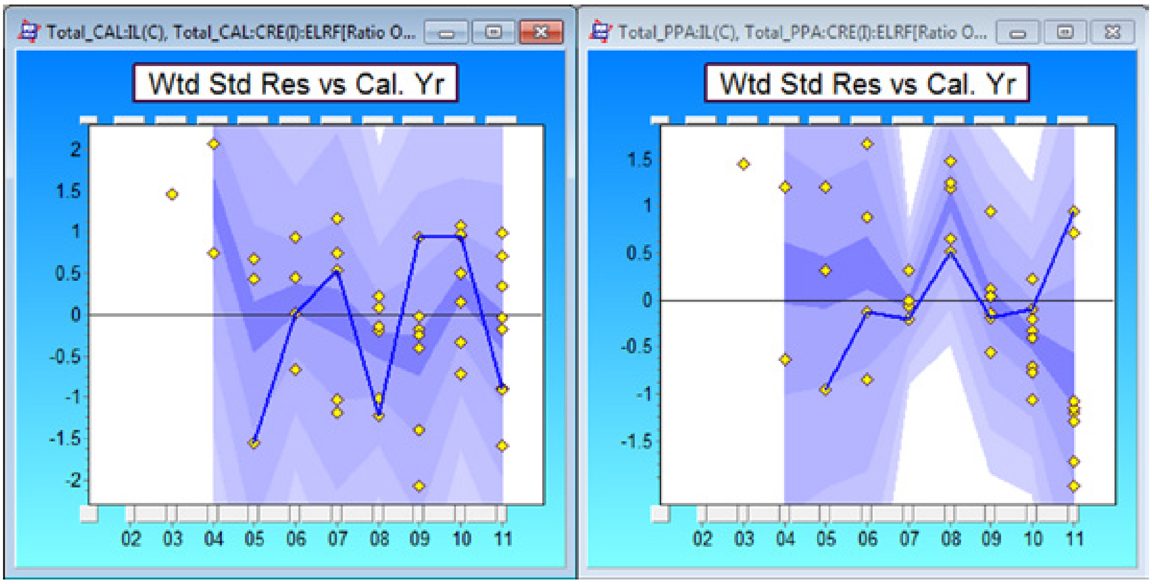
	Total_PPA:PL(I)_N	Total_CAL:PL(I)_N
Total_PPA:PL(I)_N	1	0.250
Total_CAL:PL(I)_N	0.250	1

4 iterations were executed
Residuals correlation difference tolerance 0.010%

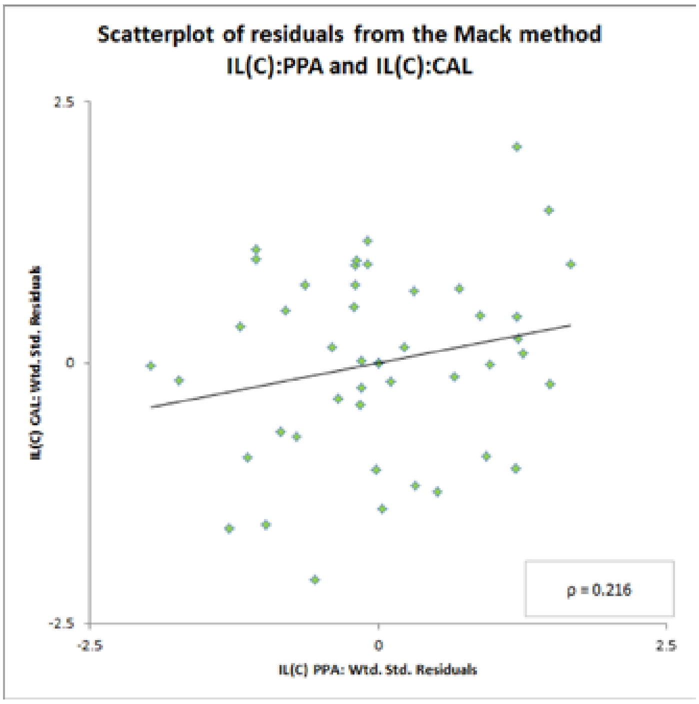
When measured, it is found to be low (0.250) and statistically insignificant (not able to be distinguished from zero correlation); indicated by the blue entry. Thus, the volatility correlation measured by calculating the correlation between the residuals obtained from the Mack method is spurious. This measured volatility correlation is a result of the lack of de-trending and does not represent correlation in the randomness (volatility).

5.2. Incurred losses: Industry PPA and Industry CAL

Similar results can arise from measuring the correlations between the residuals from link ratio methods applied to the Incurred data. However, typically the Case Reserve Estimates introduce sufficient randomness to each line of business to mask the underlying structure in the paid losses. This raises the question of whether the Incurred Losses are suitable for modelling, especially since the liability stream is unobtainable from the data.



The marked residuals (accident year 2004) for the Mack method exhibit negative correlation (by eye). This correlation is then measured and shown in the residual scatter plot of PPA vs CAL below. The negative correlation for this accident year is not realised when considering all accident years. The measured correlation for all residuals is 0.215 – much lower than the correlation in the residuals between the Mack method applied to paid losses.



The lower correlation between the IL(C) CAL and IL(C) PPA residuals when compared to the PL(C) equivalent data implies that trends and volatility in the Case Reserve Estimates are sufficient to mask the trends in the paid losses. Any trends not described in the IL(C) data are not structurally equivalent to those in the paid losses.

That there are trend changes not described by the Mack method is obvious from both residual displays above. Thus the process correlation between the two industry portfolios is still unknown. The measure of 0.215 is not meaningful as not all trends are accounted for.

6. Case study: Disaster. A consequence of blind methodology

The following two case studies represent real lines from two different companies. Each line lost millions of dollars as a result of poor reserving and pricing methodology. In contrast, the probabilistic trend family modelling framework identifies emerging experience earlier enabling effective response for both setting reserves and pricing future underwriting years.

Case study APS: Failure to identify emerging calendar year trends

A new (high) calendar year trend commences in 2006.

- Link ratio methods only respond to this trend in 2009 - **three years after** the calendar year trend commences;
- Probabilistic Trend Family (PTF) models identify the new calendar year trend in 2006 and provide indications **from 2001** that a new calendar year trend may arise;
- Actuaries armed with PTF can provide critical insight to senior management.

Case study DAD: Reserve exhaustion within three years of a 10+ run-off line.

Link ratio methods underestimate the effect of the calendar year trend.

- Initial reserve estimates from the Mack method would be exhausted within three years;
- **Significant reserve upgrades are required every year when using link ratio methods;**
- Models identified in the PTF modelling framework provide consistent reserve estimates.

Adverse development arises from poor reserving methodology **not** unusual losses.

6.1. Company APS: Worker's Compensation

In this case study, we consider real worker's compensation data to year end 2009 (after obfuscation). The company writing the line posted significant reserve upgrades over the last four calendar years.

We demonstrate the link ratio methods, particularly the Mack method, consistently underestimate the future calendar year payments. The reserve upgrades are a feature of a defective model and poor understanding of the trends in the data.

We emulate the reserving process by treating the line as being in run-off since 2005 and then step forward through the updating year-by-year comparing the Mack method with the Probabilistic Trend Family (PTF) modelling framework. In order to minimise optimisation to the data, automatic model design and forecast scenario design was applied with minimal manual intervention. As a result of the mechanical process, the PTF model used also results in reserve upgrades but with one substantial difference – the driver of the upgrade is identified to three years earlier and remedial action (pricing revisions) can be taken immediately.

Accident Years vs Development Years

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1985	1,407,300	2,487,892	1,556,672	850,044	340,956	414,920	171,922	122,672	52,857	82,803	29,947	-44,739	24,354	16,957	22,009	11,004	11,366	16,055
1986	1,505,258	2,454,522	1,407,120	790,874	399,766	240,112	227,665	111,848	21,287	48,348	14,432	11,545	7,938	7,396	31,751	20,926	5,582	-8,118
1987	1,625,945	2,503,772	1,458,173	737,114	363,887	288,821	262,482	88,937	62,238	-27,601	41,852	52,487	35,719	30,668	31,360	30,668	16,777	15,514
1988	809,996	1,254,682	739,279	404,096	227,304	84,067	78,293	126,641	49,971	20,565	-90,102	15,154	9,380	8,840	7,938	6,674	7,397	9,561
1989	756,598	1,214,994	889,372	515,944	314,437	134,217	103,370	66,928	37,523	14,252	30,307	18,483	16,056	361	14,612	15,514	9,562	20,926
1990	780,050	1,329,908	951,069	537,231	252,921	159,293	74,866	88,396	65,666	11,365	75,949	31,389	15,695	58,171	33,013	90,283	29,766	11,185
1991	780,230	1,256,847	902,180	381,727	189,600	52,136	45,641	15,334	13,169	8,298	12,809	16,055	4,510	361	-1,263	11,005	0	4,871
1992	519,913	755,695	411,312	165,968	101,566	62,238	6,674	4,150	24,173	14,252	3,067	22,730	15,514	1,444	541	721	542	721
1993	417,987	627,972	378,479	187,256	109,142	78,834	63,682	64,402	36,802	51,584	-90,643	6,354	5,231	4,871	4,691	7,216	360	
1994	734,589	1,774,053	902,902	514,682	230,551	197,899	168,674	195,182	216,300	40,770	30,849	31,831	26,338	72,160	84,788	220,449		
1995	2,794,576	5,999,924	2,742,080	1,619,992	1,063,999	768,143	539,938	294,773	273,487	171,380	142,155	323,457	575,476	130,610	312,452			
1996	9,724,282	15,815,126	8,378,859	4,387,147	2,664,869	2,234,254	1,395,575	1,037,119	582,151	605,964	522,799	647,275	666,217	501,873				
1997	16,297,156	23,860,245	11,846,146	6,328,973	4,539,586	2,830,656	1,973,576	1,615,663	1,145,901	1,246,744	908,314	1,292,566	778,246					
1998	13,958,630	15,249,573	9,419,406	5,252,346	2,853,567	2,286,750	1,879,768	1,565,872	1,864,434	1,341,094	1,128,763	1,156,003						
1999	8,690,409	11,529,003	6,652,250	4,148,840	2,891,812	2,059,626	1,130,026	1,081,137	949,265	1,181,078	782,190							
2000	9,843,346	13,950,151	7,908,556	4,680,117	3,579,497	2,178,330	1,771,889	1,798,588	1,234,477	891,717								
2001	14,981,498	20,690,437	11,558,409	7,222,314	4,923,476	4,015,704	2,768,780	2,433,235	1,883,556									
2002	21,643,670	31,079,854	17,167,405	11,272,835	7,674,938	5,403,521	3,784,767	3,069,325										
2003	30,885,128	43,570,568	27,305,705	16,828,190	11,559,851	8,138,025	5,773,161											
2004	37,999,456	57,925,177	30,995,065	19,892,708	14,625,570	10,372,458												
2005	50,098,343	63,778,498	36,620,659	26,159,082	18,988,543													

1 Unit = $1

The data are cut to accident year 2005 and five year-end triangles are created from 2005 through to 2009 as indicated by the coloured sections in the table above. Each calendar year a new diagonal is added and the Mack method reapplied to the paid losses at each year end. For each estimation, the liability stream is extracted for the years 2005 to 2009.

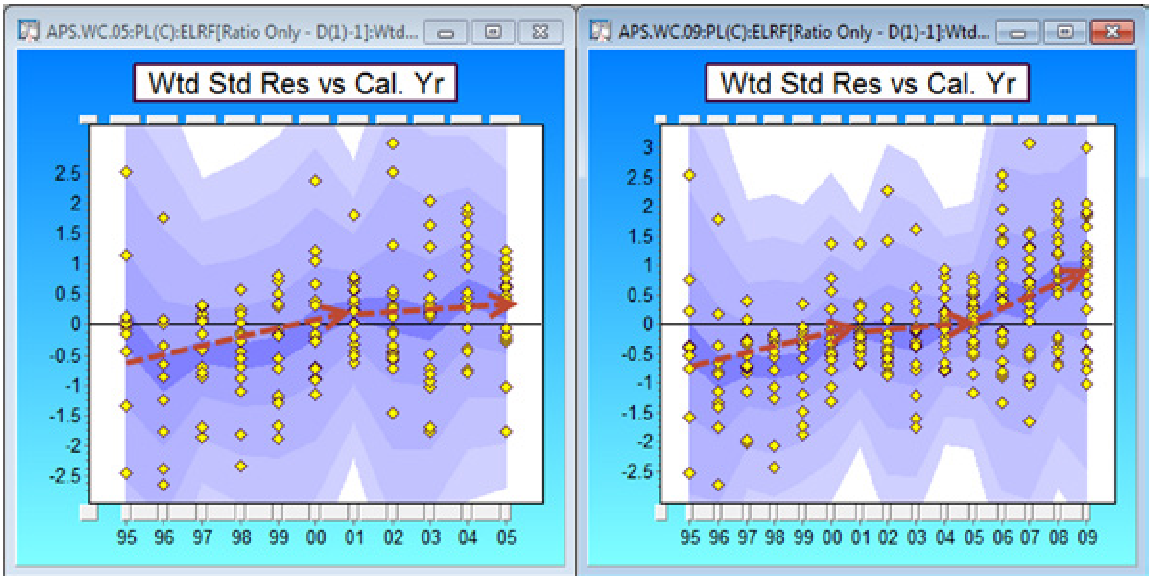
Mack method: data to year end 2005, 2006, 2007, 2008, and 2009

The following table details the Mack projections for the data for year ends 2005 through to 2009 for the losses in run-off.

Year End	Next Calendar year 2006	2007	2008	2009	Next calendar year reserve	Remainder of Reserves	Increase (Decrease)
2005	138	81	47	29	138	225	
2006	130	76	45	28	76	148	(0.53%)
2007		83	46	30	46	112	6.76%
2008			61	32	32	97	15.18%
2009				45	29	119	51.95%
	Figures in 000,000s						

There is no warning of the losses in 2007 to 2009. Initially, the method seems to be producing quite reasonable results then there are three periods of losses which are substantially higher than expected (2007 by 10% and years 2008 and 2009 by over 30%). Significant reserve upgrades ensue.

What does the method miss?



The residuals of the Mack method versus calendar year back in 2005 (left) indicate there is a definite trend in the data that is higher than the trend included in the method (remember the Mack method captures an average calendar year trend). By 2009, the method is under-predicting the most recent calendar years (right), which, if left unchecked, will result in reserve upgrades every year.

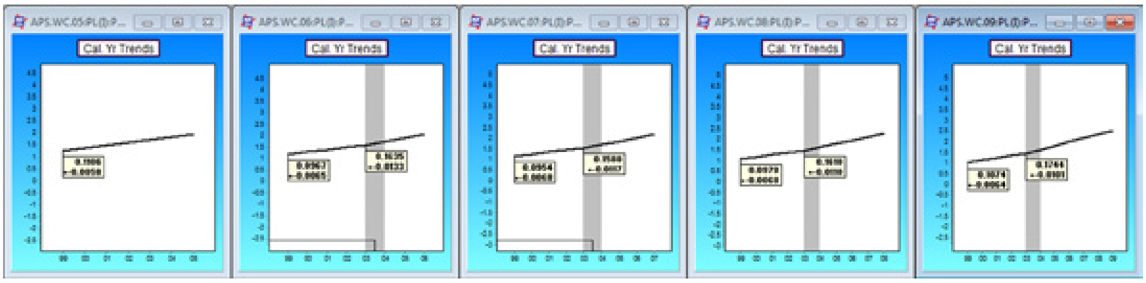
Could the Probabilistic Trend Family (PTF) modelling framework have provided an early warning system?

In order to eliminate the possibility of forecast manipulation, the following mechanical steps were applied to identify the 'best' PTF model :

- Run the modelling wizard and select the first model M1.
  - For year end 2005 there are no previous models so the modelling wizard is run to generate a base model.
  - For subsequent year ends (2006~2009), the wizard is run with the starting point being the previous year's model.
- Ensure final development trend is negative by removing any zero (or positive) development trends from the tail.
- After adjusting the development trends above, reoptimise the trend parameters.

The critical difference between the Mack method projections and the PTF projections is that the driver of the increases are clearly seen from year to year. Further, more intelligent forecast scenario creation would significantly reduce the probability of the reserve increases. In this example, the framework is demonstrated mechanically to show the applicability of the PTF framework to measure trends and provide timely and critical information to senior management.

The measured calendar year trend at each valuation year end over the 1999~2009 period are shown below. At year end 2005 there is no evidence of a change in calendar year trend between 2003 and 2004. In the next year's valuation, there is statistical evidence that the calendar year trend has increased substantially. Further, there is growing evidence that the calendar year trend is increasing (all other parameters being the same) as the estimates increase over 2007~2009.



As early as 2006, the Probabilistic Trend Family model has identified a higher trend emerging since calendar year 2003. The emergence of this calendar year trend is expected to result in reserve upgrades (from the 2005 figures). Careful monitoring and forecast scenario design can amortise the increase in reserve estimates over time according to company policy. No such amortisation was performed in this example.

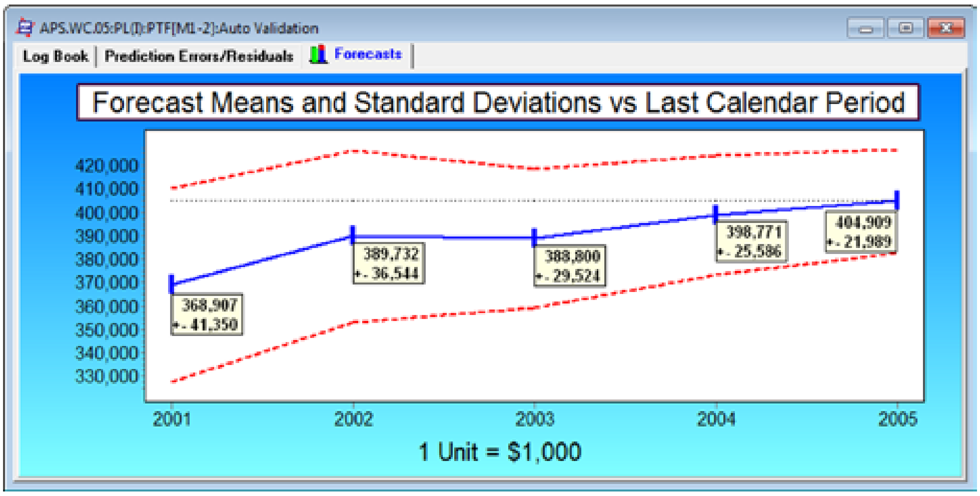
The equivalent table for PTF projections for the data for year ends 2005 through to 2009 is shown below.

Year End	Next Calendar year 2006	2007	2008	2009	Next calendar year reserve	Remainder of Reserves	Increase (Decrease)
2005	138	82	51	34	138	268	
2006	130	94	63	46	94	298	45.90%
2007		83	61	42	61	223	(4.80%)
2008			61	43	43	207	12.16%
2009				45	34	206	15.89%
	Figures in 000,000s; Model M1						

The significant reserve upgrade in 2006 is simply a result of the estimate of the calendar year trend increasing from 11.06% per year over the run-off period to 16.35% per year over the run-off period. The Mack method does not result in such a significant percentage increase in reserves (for prior years) until 2009 – three years later (and even then it is insufficient)!

The more recent revisions (2008 and 2009) are also a concern. On examining the trends for these years we find that the calendar year trend for these two years is around 6% higher than the trend since 2003. As at year end 2009, whatever conditions are driving the calendar trend inflation seem to be accelerating.

In the PTF modelling framework, adjustments to future pricing would commence well within three years of the calendar trend emerging in 2003. By the 2005 year end, there are definite signs that the more recent calendar year trend is higher than the 11%+_ estimated trend since 1994. Similarly, pricing strategy would likely be revised further in 2009 since, again the trends are again higher than expected.



The increase in reserve estimates using the model trends identified in 2005 for year ends 2001 through 2005 indicates that the more recent calendar year trends are increasing more recently. This alone would indicate to the prudent actuary using the Probabilistic Trend Family modelling framework that the forecast (in 2005) needs to be revised upward to account for the increasing calendar year trends. This is critical information required for pricing the next underwriting years (or any reinsurance).

Comparison between Mack method and PTF forecasted estimates of prior year ultimates (1995~2005) over the last five years

Mean ultimates as estimated by the Mack method each year end						Mean ultimates estimated from a selected PTF model each year end					
	Mean Ultimate						Mean Ultimate				
Acc. Yr	2005	2006	2007	2008	2009	Acc. Yr	2005	2006	2007	2008	2009
1995	16,786	17,105	17,660	17,880	18,242	1995	17,059	17,484	17,952	18,141	18,504
1996	48,112	48,681	49,233	50,065	50,775	1996	48,986	49,805	50,077	50,837	51,606
1997	72,869	74,072	74,944	76,480	77,422	1997	74,637	76,053	76,592	77,855	78,994
1998	55,003	56,571	57,797	59,226	60,486	1998	56,660	58,390	59,311	60,559	61,922
1999	39,710	40,359	40,994	42,321	43,229	1999	40,841	41,622	42,132	43,281	44,348
2000	46,319	47,417	48,649	49,818	50,740	2000	48,333	49,518	50,441	51,421	52,619
2001	68,095	70,383	72,046	74,020	75,618	2001	71,391	74,107	75,012	76,617	78,783
2002	99,997	103,112	105,655	108,336	110,379	2002	106,544	110,849	111,635	113,419	115,363
2003	142,345	146,849	151,546	156,369	160,126	2003	152,114	160,440	161,506	164,871	168,212
2004	183,414	181,653	185,688	192,720	198,348	2004	199,781	204,160	201,639	206,359	211,665
2005	243,083	220,571	218,308	227,679	237,024	2005	241,835	248,230	241,073	244,998	255,278
Total	1,067,054	1,058,251	1,074,092	1,106,922	1,134,764	Total	1,109,662	1,142,327	1,139,058	1,160,309	1,189,687
	1 Unit = $1,000						1 Unit = $1,000				

The Mack method still substantially underestimates the highlighted years especially those in accident years 2002~2005. This is due to the method failing to describe the most recent emerging calendar year trend. Since the probabilistic trend family modelling framework describes this trend and projects it into the future calendar year periods, the estimated ultimates for these years are much higher.

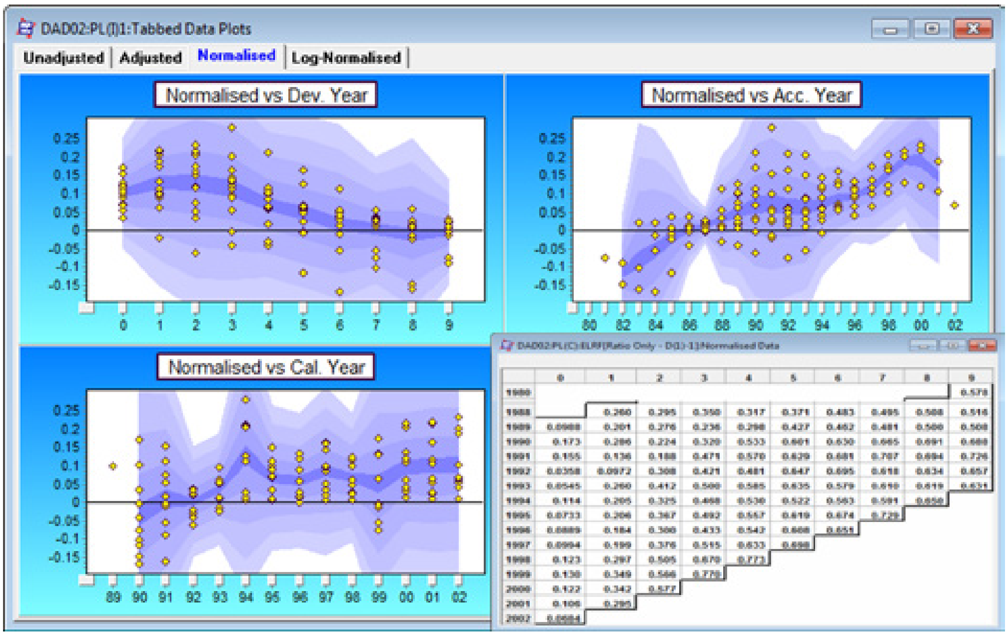
6.2. Company DAD: Adverse development resulting from a poor model

This line of business was supplied retrospectively to Insureware see if the probabilistic trend family modelling framework would identify 'adverse development'. Although not specified in the submission, it is likely a reinsurance deal had been offered on the data and, due to poor pricing, substantial capital had been lost a mere three years later.

In this example we demonstrate that there is no adverse development in the data. All the trends up to year end 2002 are sufficient to project the losses up to calendar year 2005 (when the data were supplied).

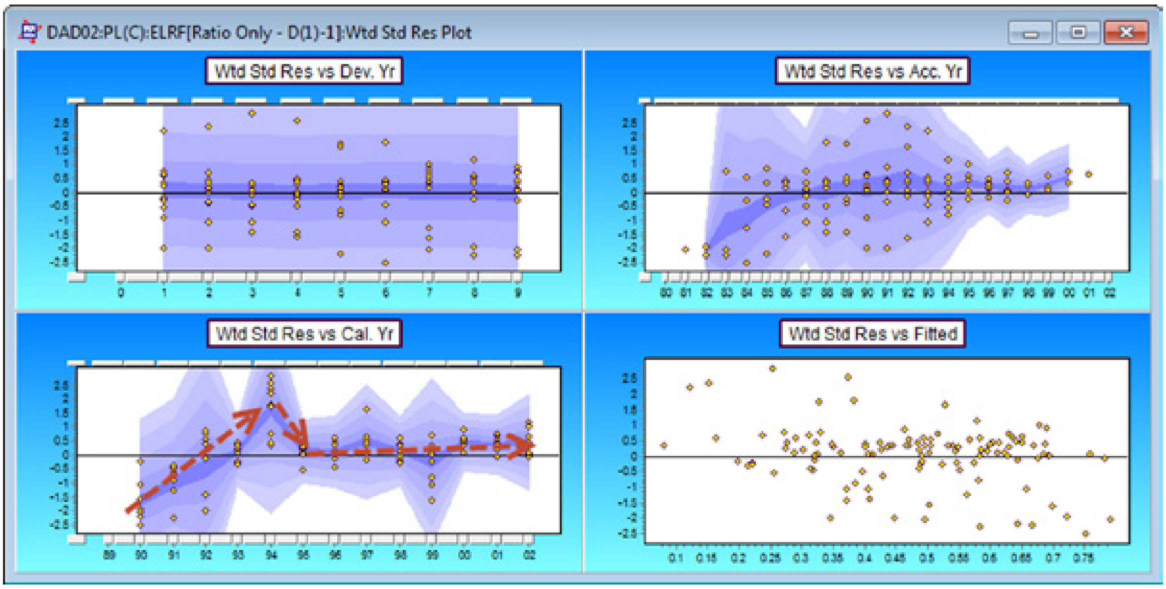
If the Mack method is used for reserving for this data, significant reserve upgrades are required from the 2002 estimates as the entire reserve allocation would be exhausted by 2005! The probabilistic trend family model is far superior.

The following real (normalised) data available to calendar year 2002 and development period nine. Calendar year trends are present as indicative in both the accident year and calendar year directions. Further, the development trend decrease seems to be slowing.



The data exhibit changing calendar year trends and a simple development trend structure.

Mack method: **data to year end 2002**



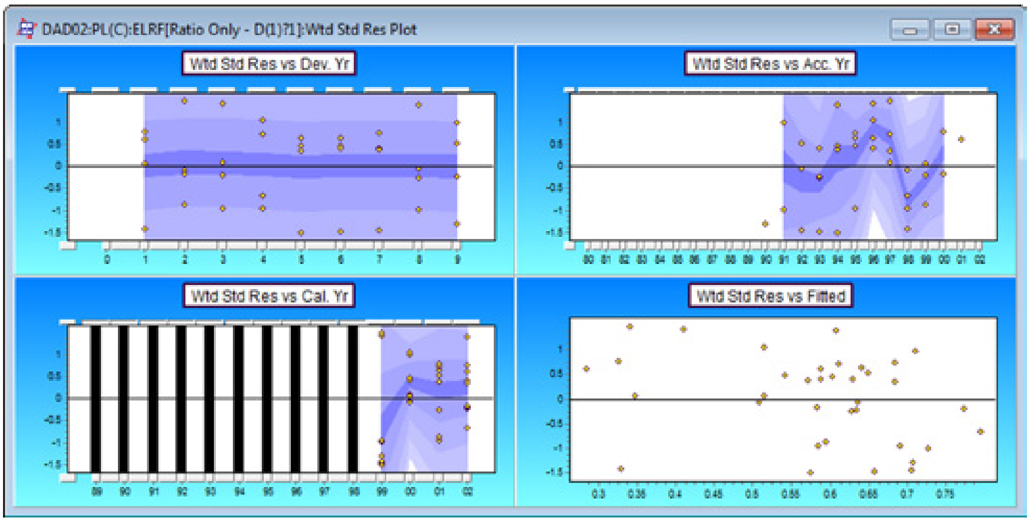
The Mack method does not capture the trend structure in the data. The red arrows mark the obvious calendar year trend structure not described. The net effect of the missed calendar year trends is that the method under-projects the next calendar years.

Mack method: **reserve exhaustion within three calendar years**

The Mack method estimate of the total reserve as at year end 2002 is 3.5M. The total calendar year losses over 2003~2005 exceed 4.0M. Considering the run-off period is 10 years, exhaustion of capital after three years (assuming no reserve upgrades) is a very poor result.

What if instead the volume weighted average over the last four calendar years was used? Does this link ratio method provide a better result? The residuals (below) from the last four years seem a superior fit.

If the Mack method was used across all calendar years, the loss is severe at over $1.2M relative to the Mack method estimates for these three calendar years in 2002. Further the reserves would have been exhausted by 2005 if no upgrade was made from the Mack method estimate in 2002.



Weighted average last four calendar years – still significant upgrades

Assuming the actuary presented the figures on the right hand side and these corresponded to the booked reserves. How well does the actuary compare to the real data – available to calendar year 2005? The Mack method applied to the last four calendar years is reapplied every calendar year from 2002 till 2005.

The reserves are split between the reserve for the next calendar year (for the valuation period) and the reserve allocated to the remaining period. The split facilitates tracking of relative reserve increases (decreases).

Year End	Next Calendar year			Next calendar year reserve	Remainder of Reserves	Increase (Decrease)
	2003	2004	2005			
2002	1,453	1,214	870	1,453	3,484	
2003	1,579	1,336	1,053	1,336	3,595	41.53%
2004		1,401	1,061	1,061	2,634	2.78%
2005			1,069		2,649	0.57%
	Figures in 000s					

The most influential losses are the next calendar year. These losses are under-projected (by $126k) with the result that the mean reserve (for the remaining calendar periods 2004 onward) is increased by 41.5% in the next valuation period – the increase of $1.4M is close to paid losses in the 2003 calendar year!

This is not a small reserve upgrade.

Subsequent reserve upgrades occur every year thereafter (though with decreasing magnitude). In the years 2003~2005 losses exceeded the 2002 projections by $386k.

Could disaster have been avoided?

The total mean reserves projected by the Mack method in 2002 was $3.5M. As seen from the table above, the total losses between 2003 and 2005 exceeded $3.5M. The line of business would be in distress as a result of a poor model.

Similarly, if the Mack method applied to the last four calendar years was used in 2002, the estimated total mean reserves is $4.9M. By the end of 2005, only $0.9M would be available to cover the remaining liabilities. Even based on this method applied at year end 2002, over $1.4M is required to pay the losses in the calendar years 2006~2011.

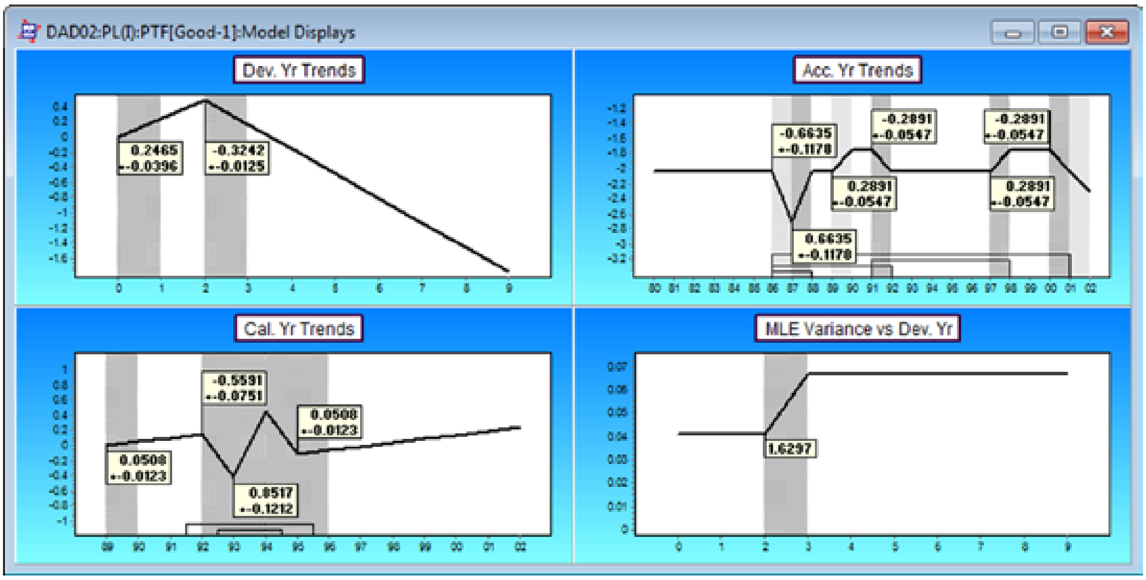
The key feature of the issue, as detailed previously in this brochure, is that the method is not able to describe the salient features of the data. As a result, poor feedback is provided to the managerial team resulting in both poor decisions regarding booked reserves and pricing future underwriting risk. The modellers do not understand the risk of the line, so how can the people relying on the model results gauge the value of the projections?

Analysts must use the right tool for understanding trends in the business so as to be best placed to provide information that is timely and accurate.

The Probabilistic Trend Family (PTF) modelling framework is a critical tool for any analyst to have in order to detect trends, volatility, and assess future risk. This tool measures the trends in the three time directions (development, accident, and calendar) along with the volatility around those trends. If this tool was used by the company, would the results have been any different?

Probabilistic Trend Family model

In order to minimise influence of designing a model to fit the 2005 data in 2002, the automatic modelling wizard was applied to the data. The only modification to the modelling wizard model was to add the final accident year level change to decrease by the same magnitude as the previous accident year as the modelling wizard does not add parameters for a single observation).



Completing the square using the trends measured from the data results in projections for 2003~2005 of: $1.56M, $1.40M, and $1.05M. The total mean reserve is $6.3M – more than 28% higher than the $4.9M mean reserve projected by the Mack method applied to the last four calendar years.

Year End	Next Calendar year 2003	2004	2005	Next calendar year reserve	Remainder of Reserves	Increase (Decrease)
2002	1,556	1,395	1,053	1,556	4,745	
2003	1,579	1,470	1,131	1,470	3,648	7.86%
2004		1,401	1,143	1,143	2,625	3.29%
2005			1,069		2,574	(1.94%)
	Figures in 000s					

Note no changes were made to the structure of the model shown previously, rather the structure was applied as described in 2002 to each subsequent year as no change in trends were found as new data arrived in calendar year time. The increase in reserves for 2003 and 2004 are are reflection of the uncertainty of the final accident year level in the 2002 model and minimal analysis of the trend structure in 2002~2005 to ensure the valuation process was fair. That is, since in the PTF modelling framework all future trend assumptions can be modified, care was taken to separate the year end valuations so it was impossible to select future trends (in 2002 say) based on knowledge of the future calendar years (2003~2005).

Summary

For this line, if the PTF modelling framework was applied with appropriate recommendations for managers then, assuming the projections were booked 'as is', then the difference between the methodology is clear.

- Link ratios methods: significant reserve upgrades, initial estimates out by over 30% (assuming the more conservative last four calendar year model is applied).
- Probabilistic Trend Family: small reserve increases for 2003 and 2004 consistent with the measured calendar year inflationary trend.

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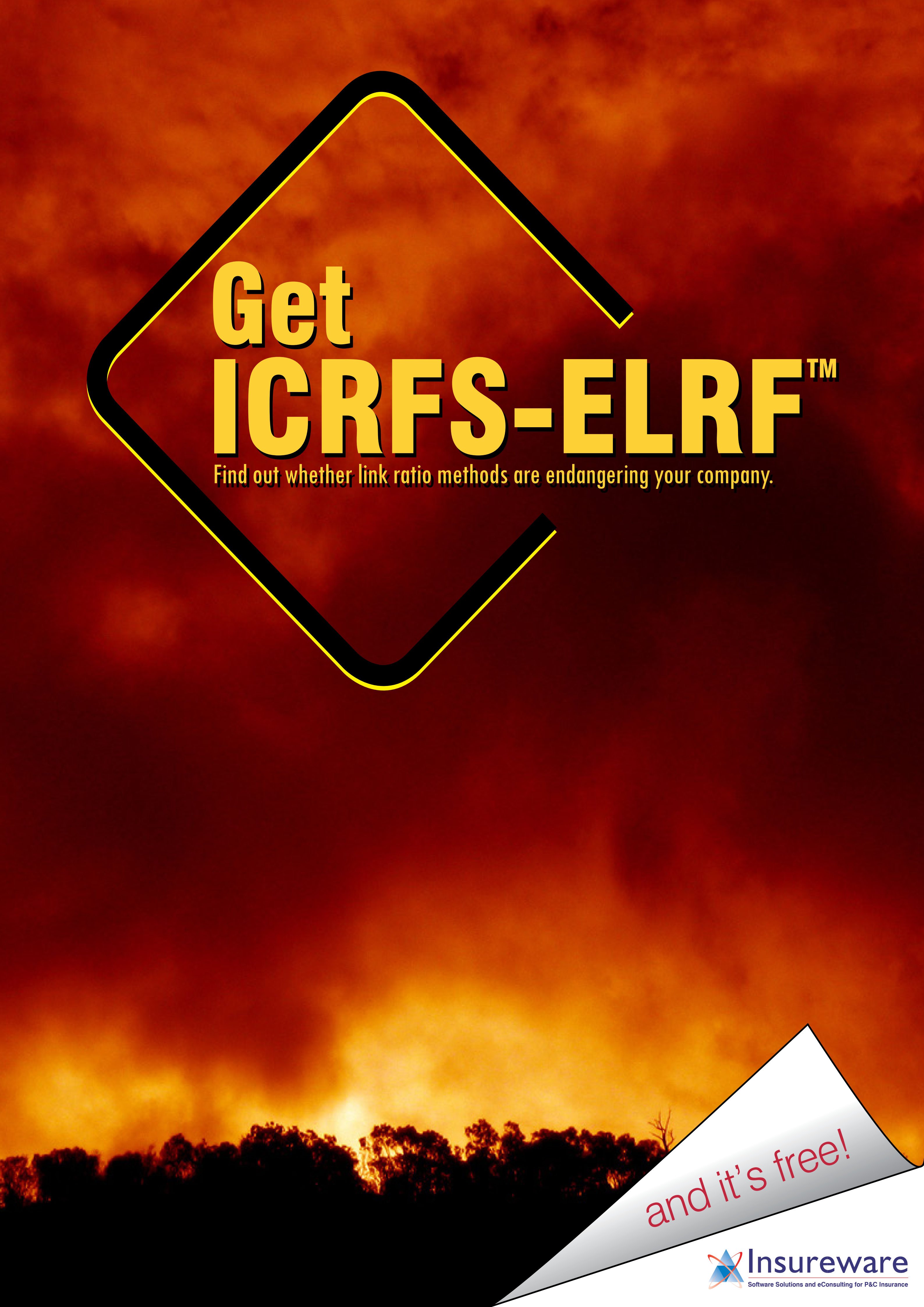
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